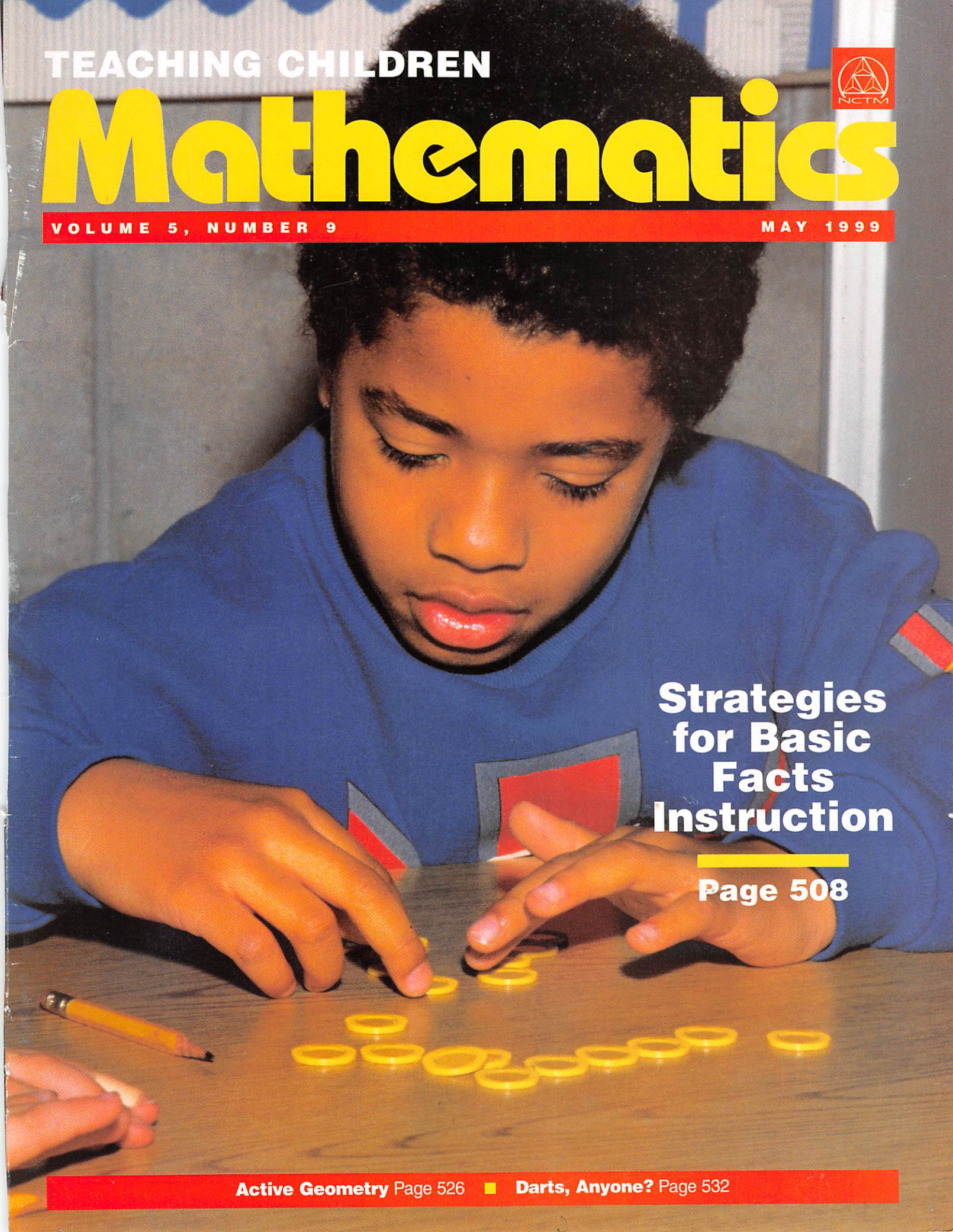
TEACHING CHILDREN

Mathematics

VOLUME 5, NUMBER 9 MAY 1999



Strategies for Basic Facts Instruction

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Call for Manuscripts for the February 2001 Focus Issue:

Mathematics and Culture

The February 2001 issue of *Teaching Children Mathematics* will focus on the role of culture in describing, learning, teaching, and assessing mathematics. Mathematics is a vital aspect of culture. It reflects and is simultaneously created with aspects of the cultures of all its users. Teachers can help students gain a deeper understanding of how mathematics has evolved and continues to evolve and of the students' role in this evolution as they connect mathematics and culture in the classroom. As teachers guide, facilitate, and extend students' investigations and experiences, multiple concepts are simultaneously explored. When students realize that they possess an innate ability to think and to become creatively involved in the study of mathematics, they learn that it is everywhere and part of everything in which they participate.

The Editorial Panel wishes to illustrate the importance of connecting students' mathematics study and culture by using examples of mathematical contributions from around the world, culturally based mathematics in both Western and non-Western settings, informal or out-of-school mathematical situations, social connections and implications, and assessment of diverse learners. Manuscripts that include photographs and examples illustrating cultural diversity are especially encouraged.

The list of topics and related questions are intended to guide authors in addressing one or more aspects of mathematics and culture. Articles that discuss the diverse cultural needs and unique experiences of students as related to race, ethnicity, gender, social class, and ability or disability are particularly welcome.

Mathematics of People throughout the World: Ethnomathematics in the Classroom

- What are examples of culturally influenced mathematics used by people throughout the world?
- What are some examples of the cultural contributions made by famous mathematicians from around the world?
- How can games played in other cultures be helpful in learning basic mathematical skills, thinking strategies, and problem solving?

Culturally Based Mathematics Curriculum and Instruction

- What examples arising from children's everyday lives have been addressed by culturally based mathematics instruction using historic tools or current technologies?
- How have culturally relevant mathematics curricula improved the mathematics learning of all students?

Folk Math: Informal or Out-of-School Uses of Mathematics

- How is mathematics used by people in informal or out-of-school situations?
- How can teachers enrich children's mathematical understanding?
- What do children gain from the study of examples of informal or out-of-school situations?

Assessment of Diverse Learners

- How do we attempt to understand the multilingual and multicultural mathematical differences of our students? What are some examples?
- How can the assessment of children's thinking and dispositions be used to enhance instructional planning?

Mathematics and Social Issues

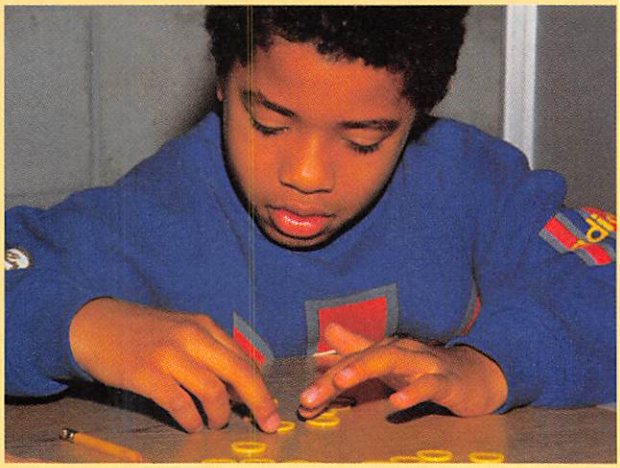
- How might the analysis of social issues improve students' mathematical, social, and cultural understanding?
- How can students use social issues to reason about, see, model, or justify mathematical ideas?

Six copies of the completed manuscript should be sent for review to *Teaching Children Mathematics,* 1906 Association Drive, Reston, VA 20191-1593, by **30 November 1999.** All manuscripts should be double-spaced and should indicate clearly that they are being submitted for the February 2001 focus issue. Author identification should appear only on a cover page. ▲

TEACHING CHILDREN
Mathematics

VOLUME 5, NUMBER 9 MAY 1999

ON THE COVER



Helping students master basic number facts in various ways without resorting to drilling and timed tests is the subject of "Strategies for Basic Facts Instruction" by Andrew C. Isaacs and William M. Carroll. Photograph by Robert D. Davis; all rights reserved. Readers are encouraged to submit color photographs or color slides of children actively involved in exploring mathematical ideas for possible use on the cover, accompanied by a brief explanation of the activity. Please send such submissions to the "Readers' Exchange" department, *Teaching Children Mathematics,* 1906 Association Drive, Reston, VA 20191-1593. Include a statement from the parents of the children pictured, indicating their permission for NCTM to publish the photograph, and include an appropriate credit line for the photographer.

The **mission** of the National Council of Teachers of Mathematics is to provide vision and leadership in improving the teaching and learning of mathematics so that every student is ensured an equitable *Standards*-based mathematics education and every teacher of mathematics is ensured the opportunity to grow professionally. **Teaching Children Mathematics** is an official journal of the National Council of Teachers of Mathematics. It is a forum for the exchange of ideas and a source of activities and pedagogical strategies for mathematics education pre-K–6. It presents new developments in curriculum, instruction, learning, and teacher education; interprets the results of research; and in general provides information on any aspect of the broad spectrum of mathematics education appropriate for preservice and in-service teachers. The publications of the Council present a variety of viewpoints. The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of the Council.

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Correspondence should be addressed to *Teaching Children Mathematics,* 1906 Association Drive, Reston, VA 20191-1593. Manuscripts should not exceed ten pages of text and should be typewritten on one side only, double-spaced with wide margins. Figures should appear on separate sheets. Manuscripts should be prepared according to the *Chicago Manual of Style* and the *United States Metric Association's Guide to the Use of the Metric System.* No author identification should appear on the manuscript; the journal uses a blind-review process. Five copies are required. Three disks for computer programs with more than fifteen statements should be furnished. **Permission to photocopy** material from *Teaching Children Mathematics* is granted to persons who wish to distribute items individually (not in combination with other articles or works), for educational purposes, in limited quantities, and free of charge or at cost; to librarians who wish to place a limited number of copies on reserve; to authors of scholarly papers; and to any party wishing to make one copy for personal use. Permission must be obtained to use journal material for course packets, commercial works, advertising, or professional-development purposes. Uses of journal material beyond those outlined above may violate U.S. copyright law and must be brought to the attention of the National Council of Teachers of Mathematics. For a complete statement of NCTM's copyright policy, see the NCTM Web site, www.nctm.org. For information on **reprints or back issues,** write to the Customer Service department in the Headquarters Office. Information is available from the Headquarters Office regarding the three **other official journals,** *Mathematics Teaching in the Middle School,* the *Mathematics Teacher,* and the *Journal for Research in Mathematics Education.* Special rates for students, institutions, bulk subscribers, and retired members are also available. The **index** for each volume appears in the May issue. *Teaching Children Mathematics* is indexed in *Academic Index, Biography Index, Contents Pages in Education, Current Index to Journals in Education, Education Index, Exceptional Child Education Resources, Literature Analysis of Microcomputer Publications, Media Review Digest,* and *Zentralblatt für Didaktik der Mathematik.*

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Readers' Exchange

Ethnomathematics

Thank you for including Claudia Zaslavsky's "In My Opinion" article on "Ethnomathematics and Multicultural Mathematics Education" in your May 1998 issue (pp. 502–3). More articles are needed that address how mathematical concepts and ethnomathematical perspectives are, or might be, synthesized in the mathematics curriculum in ways that are sensitive to all cultures. Do exemplars exist in curriculum, among teachers at the different grade levels, in mathematics textbooks, and so on? How might we begin?

Zaslavsky's information is a good beginning. We need to continue delving into this topic, which involves a plethora of considerations, complexities, challenges, and budding awareness for the entire educational community.

Mary Jo Cittadino
Emeryville, CA 94608

The Editorial Panel responds: The February 2001 focus issue will explore the theme of "Mathematics and Culture." We encourage readers to share their ideas. See page 499 for the "Call for Manuscripts" for this issue.

More on the Three Pigs

I would like to comment on the article "A Teacher's Journal: Hands-on Addition and Subtraction with the Three Pigs" by Mary Marron Bartek (October 1997, 68–71). Although many good ideas appear in the article, I think the author imposes an algorithm for addition and subtraction on her students, along with too many contrivances and rules.

To me, using the Three Pigs to generate large numbers is rather forced and might tend to detract from the meaning and enjoyment of the tale for students. More conducive to working with large numbers would be to use the "Pied Piper of Hamelin" poem by Robert Browning. Even better is the rhyme "As I was going to St. Ives," which actually ends with a "How many?" question. Tying mathematical ideas to stories can be very powerful, but the ideas should flow naturally from the story.

I find that ideas are very commonly set forth as rules by teachers and textbooks. Perhaps teachers think that students will not learn them, or not quickly enough, unless they are given or imposed. Consider the rule that no pig is allowed to have more than nine bricks. I do not doubt that this idea should be learned. I question only whether it should be called a rule and thus obeyed or whether it just might be termed an idea that can be efficient, helpful, and sometimes necessary. Significantly, the same rule—no pig allowed to have ten bricks—is to be broken routinely in the subtraction part whenever regrouping is needed. Students need to know when to use it. Why call it a rule?

Nor do I like the idea of having the class all do things in the same way, introduced by the three children/pigs at the front of the room, as directed by the teacher. I would rather pose a problem to the class as in the article (i.e., 258 + 473) but let the children attack it first, as individuals or groups, and then look at what they have done and how. Students will thus show their thinking, including ideas about addition, place value, written notation, and oral language. The teacher will be able to learn much more about what and how they are thinking and be able to help them make better connections between where they are and where she wants them to be than if they are all fed the same instructions and assumed to understand them on the basis of their successful repetition of the correct answer.

Students should take on more responsibilities, and, to effect that outcome, teachers should do less. What purpose is served by the teacher's so studiously coming up with a sequence of variations? Why must the teacher come up with the numbers, make all the drawings, and fashion the problem sheets? Students should come to own their activities and ideas.

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The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Readers who are commenting on articles are encouraged to send copies of their correspondence to the authors. Because of space limitations, letters and rejoinders from authors are limited to 250 words each. Letters are also edited for style and content. Please double-space all letters that are to be considered for publication.

Bartek responds: "The Three Pigs" story is old and familiar to second and third graders and as such is one that they feel comfortable toying with. Nothing in the article suggests that this story is the only one that might logically be used to address large numbers. You suggest several valid choices. However, the strategy of three pigs' dealing with three place values and working in building supplies is not too great a leap for my students.

You mention your dislike of the word *rule* in setting up the processes where the students will pass their sets of ten to the next pig. I use the term "rule" interchangeably with "code of conduct," "standard of behavior," or "the way we do things here." Should you decide to try the lesson, you may choose some other word.

Another concern involved the contrived variations in the problems. The pigs lesson can help students to physically solve so many variations of three-digit problems—subtracting across one or more zeros, for instance—that being sure to include these scenarios increases the opportunity for learning. A built-in extension for students who have a greater understanding is to request that someone create a problem where a particular trade will have to take place.

I believe that sharing ideas teacher to teacher is a good way for all of us to learn. But what works for me, in my school, is not necessarily what will work for everyone. If you find some kernel of an idea in my article, it is there for you to explore. If changing something makes this lesson work better for you, then go for it.

Correction

The Zome System was reviewed in the February 1999 issue of *TCM* on page 386. The telephone number for further information should have been (888) 966-3386. We regret this error. Additionally, the price for the Zome System has been reduced to $124.95.

In NCTM journals

Readers of *Teaching Children Mathematics* might enjoy the following articles in the January 1999 issue of the *Journal for Research in Mathematics Education:*

- "Relationships between Research and the NCTM Standards," James Hiebert; www.nctm.org/jrme/abstracts/volume_30/vol30-01-hiebert.html
- "Gender Differences in First-Grade Mathematics Strategy Use: Parent and Teacher Contributions," Martha Carr, Donna L. Jessup, and Diana Fuller

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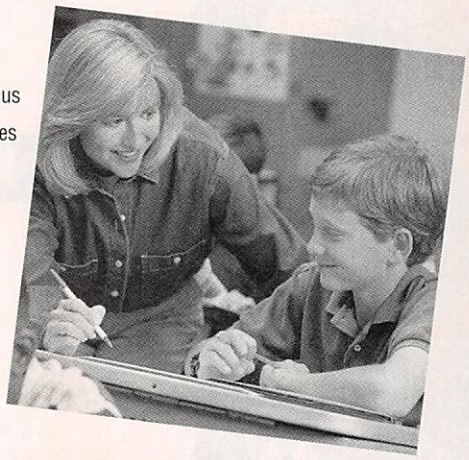

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A Teacher's Journal

What's in the Box? A Problem-Solving Lesson and a Discussion about Teaching

Elliott Bird

While holding up a small, thin, cardboard box, I asked, "What do you think is inside?" As I shook it, my class could hear the sounds of objects shuffling about. Puzzle pieces, playing cards, candy, and pencils were some of the responses that I received. "They are puzzle pieces of a sort," I said, "because this is a puzzle and I will show you a piece only after you have identified it. What do you think is in the box?"

S: Candy.
T: No, there's no candy.
S: Are they all the same color?
T: What color do you have in mind?
S: Red.
T: There are some red things in the box, but there's more than one. Which one would you like me to take out?

Photograph by Emily G. Schreiber; all rights reserved

In this way, my students learn that identification involves color and more. They start guessing. Before long, they have discovered that the "things" in the box are cardboard pieces distinguished by size, color, and shape. To identify a piece, they have to name each of its attributes, for example, a large red circular shape. I show that piece to them and leave it on display for the remainder of the activity.

S: Is there a small red circle, too?
T: Yes, here it is.

My implicit message is that the work of classifying is theirs. My role is to answer questions that ask for a piece with all attributes identified. The collection that I use has two sizes—small and large; four colors—red, blue, green, and yellow; and three shapes—square, circle, and triangle; twenty-four pieces in all. To determine all the pieces, students need to use ingenuity and to take risks.

Elliott Bird, bird@titan.liunet.edu, is a mathematics professor at the C. W. Post Campus of Long Island University, Brookville, NY 11548. He visits a half dozen or more middle school classes every week to model constructive teaching and learning principles.

In this activity, we refer to the circular shape as a circle, the triangular shape as a triangle, and so on. A piece itself cannot be a circle, triangle, or square. But the shape of its base *is* that of a circle, triangle, or square. When we identify a piece as a circle, we are referring to one of its attributes—shape. Later schooling refines the distinction between a circle and an object with the shape of a circle.

Different students ask different questions, and different classes proceed differently. But soon, perhaps eight pieces are on display, say, two red circles, small and large; two blue circles, two red squares, and two blue squares. I place the pieces randomly on the display so that to see patterns, students have to sort mentally.

Visualizing patterns is integral to this activity. Usually I emphasize the existence of patterns. But an important element of the problem solving here is letting students become aware of their value. Making the patterns obvious by sorting the pieces as I display them removes the pleasure of discovery. It also sends a subtle message that the mathematics is so complicated that the teacher has to make it obvious. Students themselves find many ways to organize the pieces and, in sharing their findings, learn to appreciate different approaches. In determining how much to help students, I weigh their ability to progress without assistance against their likelihood of being stumped. The more likely they are to be able to do it themselves, the less likely I am to assist.

Determining all the pieces requires sorting. In the early questioning, mental sorting is already taking place. Had candy been in the box, ensuing questions would have focused on categorizing and identifying the candy. I do not do the focusing for the students by sorting the displayed pieces. I set up the challenge and guide them through it, resisting their attempts to have me simplify the task.

I have played this game many times with youngsters from kindergarten through sixth grade and even with adults. Every group has succeeded in getting all pieces out of the box, and I have found that the more I hold back, the greater are their pleasure and shared enthusiasm. Students enjoy the challenge of mentally sorting the pieces to determine what remains in the box.

S: Do you have any other colors?

T: What do you have in mind?

S: Orange.

T: If there are orange pieces, there are more than one. Which one do you want me to take out?

S: The large one.

T: If there is a large orange piece, there is more than one.

S: Take out the large orange circle.

T: Oh, I'm sorry; I don't have a large orange circle. Who has a piece in mind they'd like me to remove from the box?

For some games, such as "guess my number," the objective is to solve a problem with a minimal number of questions. Here the focus is on exploration, relationships, even precision of language. I do not want to hurry the revelations. If they come from the students at a leisurely pace, they will be more meaningful and better retained. When students do raise, however obliquely, a relevant or important issue, I can reflect it with a question while calling attention to classification. That is why, when asked whether all pieces are the same color, I ask back, "What color do you have in mind?" Once students have determined a color in the set, I can steer the questioning to other attributes.

Taking Risks

The student who asked whether I have any other colors is taking some risk, perhaps small. Instead of following along the lines of previous successful inquiries—such as those that revealed the red and the blue pieces—this student is opening the possibility of other colors. Such risk taking develops naturally in this activity. Outspoken students offer suggestions from the start; more reticent ones participate as they see patterns. For these students, just speaking out involves risk. I have enough opportunities to raise questions to involve everyone. An important part of my role is to stimulate the class to get total participation.

I do not draw attention to the risk taking. Such attention creates self-consciousness and, possibly, tension, and is counterproductive. If I talk about taking risks at the same time that I am trying to generate such behavior, then urge it on my students and congratulate them for it when they speak out, the action takes on a different meaning and a different value. It is not an independent, student-initiated action but is engendered by my urging. Their area of concern becomes larger than I intended. Some of their efforts are aimed at pleasing me—a different, unspoken task from the one on which I want them to focus.

So I appear to ignore the risk-taking aspect of the question whose format ("Do you have any other colors?") differs from the acceptable form ("Do you have a large orange circle?"). Drawing no special attention to this question, I respond in the same way that I respond to all questions. I want to avoid my students' becoming dependent on me for the gratification of their experimental impulses. I take all their questions seriously,

betraying neither pleasure nor disappointment. This manner of responding creates an atmosphere free of judgment that produces a sense of security and comfort. So assured, students shed inhibiting self-doubts and become willing to experiment and to make conjectures. In this way, both their mathematical capabilities and their self-confidence grows. And this kind of risk taking—trying out new ideas—becomes an integral part of their problem-solving strategies.

Guiding the Activity

When the student asks whether other colors are in the box, I reflect the question ("What do you have in mind?"), reasserting my role as prodder and stimulator but not as too much of a resource. To the extent possible, I want students to be the sources of ideas. When asked for a large orange circle, I only say no, therefore not excluding other orange pieces. Some students need to hear that no small orange circle, no large orange triangle, and so on, is present, whereas others are already considering other colors and shapes. I continualy monitor what is going on in the classroom and guide the lesson accordingly.

We do not talk about *whether* a square is a rectangle; it is

S: Do you have any yellow pieces?

T: If I do, I have several. Which one would you like me to take out?

S: Take them all out.

T: Oh, I bet you'd like that. Then there would be almost no puzzle to solve. Sorry, you'll have to name a single piece that you'd like me to remove.

S: How about a large yellow rectangle?

T: I have a large yellow rectangle. Here it is.

I remove and display the large yellow square.

S: That's not a rectangle; that's a square.

T: You're right. It is a square. But it is also a rectangle. Do you know why a square is a rectangle? Does anyone know?

Here is another of many teachable moments in this activity. I have treated this issue in different ways. We do not talk about *whether* a square is a rectangle; it is. We might discuss *why* a square is a rectangle. If time is short, I may leave the question open. On occasion, I might even explain why, especially if we will not have another opportunity for discussion, for example, when I work with in-service teachers. The word *rectangle* has two parts: *rect,* from the German word meaning *right,* and *angle.* A rectangle has only right angles. Any four-sided figure with only right angles is a rectangle.

I prefer discussion to "telling." It creates a sense of mathematics as a dynamic, living subject, something that is appropriate to talk about, even argue about. I like discussions best when I can step back while students talk. But I keep in mind the significance of their topic. We should know that squares are rectangles, and we should know why. Because other issues in mathematics are more substantive, I keep the discussion short.

S: You must also have a small yellow square.

T: You're right! How did you know that? [as I displayed the piece]

S: Well, the pieces seem to come in twos. There's a large and a small version of each piece we've seen so far. I figured there'd be a large and a small yellow square, too.

T: Ah! So you see a pattern. There are lots of patterns here. Does anyone else see a pattern? Can you use your pattern to find another piece that's still in the box?

S: Do you have any rectangles that are not squares?

Responding to Students

Every classroom comment that I make involves a judgment call. Education is about decision making. Just as I am concerned about helping my students make considered, useful decisions, I, too, am always deciding how to handle a question or comment. The occasion and the exigencies of the moment determine my response. I want to maintain students' interest and enjoyment. In response to the question, "Do you have any rectrangles that are not squares?" I might ask, "What do you have in mind?" but I might also say, "All rectangles in my collection are squares." I intend this comment to generate another question, and I wait for it. It may be general "Do you have any triangles?" but the students probably know now to identify a piece with three attributes.

S: Do you have a big red triangle?

T: Yes, I have a big red triangle."

Now the questions come more quickly, and more students feel the confidence to participate, perhaps because they do not find my neutrality threatening. By this point in the activity almost all hands are up. We get a small red triangle, and two each—small and large—of blue and yellow triangles. Then, with third graders and up, I interrupt the process.

Following Up

By this point in the activity I have gone about as far as I can while maintaining student interest and participation. Little remains but to complete patterns and determine the remaining pieces. For some students, that challenge is enough because the pieces are displayed randomly and they need to sort mentally. With kindergarten, first-grade, and most second-grade children, we stop when we get all the pieces out of the box. In subsequent lessons, I follow up with sorting and patterning activities.

For example, I might give each child or pair a full set of twenty-four attribute blocks and ask them to do something with the pieces. Some create pictures, others sort in different ways: by size, color, or shape; in rows, columns, or lines. Then the students look at one another's creations. Afterward, we discuss their creativity and their patterns.

We also play a game with all pieces displayed—randomly, as when we first took them out of the box. Students cover their eyes, and I remove one piece. They have to figure out which piece that I have removed. At other times, I hide a piece in my pocket. In the style of Twenty Questions, they have to determine the hidden piece. Then, in groups of two to four, they play these games, taking turns being teacher. I also encourage them to invent other games that involve them in asking one another higher-order-attribute questions.

With third graders and higher, I choose activities that lead them to think carefully about how to count the pieces. After only eight to ten pieces have been removed from the box, I interrupt the game and ask, "How many pieces do you think are in the box altogether?" I record students' guesses on the chalkboard. As more and more pieces are removed from the box and put on the display, I encourage additional guesses and allow changes in prior guesses. "You're always allowed to change your guess when I'm teaching," I say. I hope that students will use the same patterns that they have been using to find missing pieces to help them determine the total number of pieces.

The activity finishes when students realize that all twenty-four pieces are out of the box. Then we return to the numbers on the chalkboard, and I ask students whose guesses are closest to twenty-four to explain the reasoning that they used to arrive at their number. In a subsequent lesson, we consider this question in more detail. Students generate number sentences that describe the full collection, for example, $12 + 12 = 24$ or $2 \times 12 = 24$, sorting by large pieces and small. In fact, every factorization of 24 into any number of factors can be represented with these pieces. I describe this part of the activity in greater detail in another article, "Counting A-Blocks: Constructing Meaning for the Multiplication Principle" (Bird, in press). It offers a nice illustration of the fundamental principle of counting.

Extending the Activity

This activity is only the beginning of a rich series of problem-solving activities that use attribute materials. Posing Venn-diagram situations, first with one ring then with two nonoverlapping rings, leading to two and even three overlapping rings, helps students determine how to classify pieces. For example, drawing a single ring and labeling it, say, "big," students learn that all big pieces must be placed *inside* the ring; all others belong *outside.* With two nonoverlapping rings, students might label one with a triangle and the other with a square then alternately place the pieces, determining that circles belong outside both rings. With a red ring drawn on the chalkboard, the teacher might hold up blue chalk and say, "This red ring is for all the large pieces. Who will come up and draw a blue ring for all the triangles?" A trial-and-error approach leads students to discover the need for overlapping rings. Then, with two attribute labels hidden from view, teams of students play a game, trying to identify the correct attribute for each ring by placing pieces.

I encourage students to ask one another higher-order-attribute questions

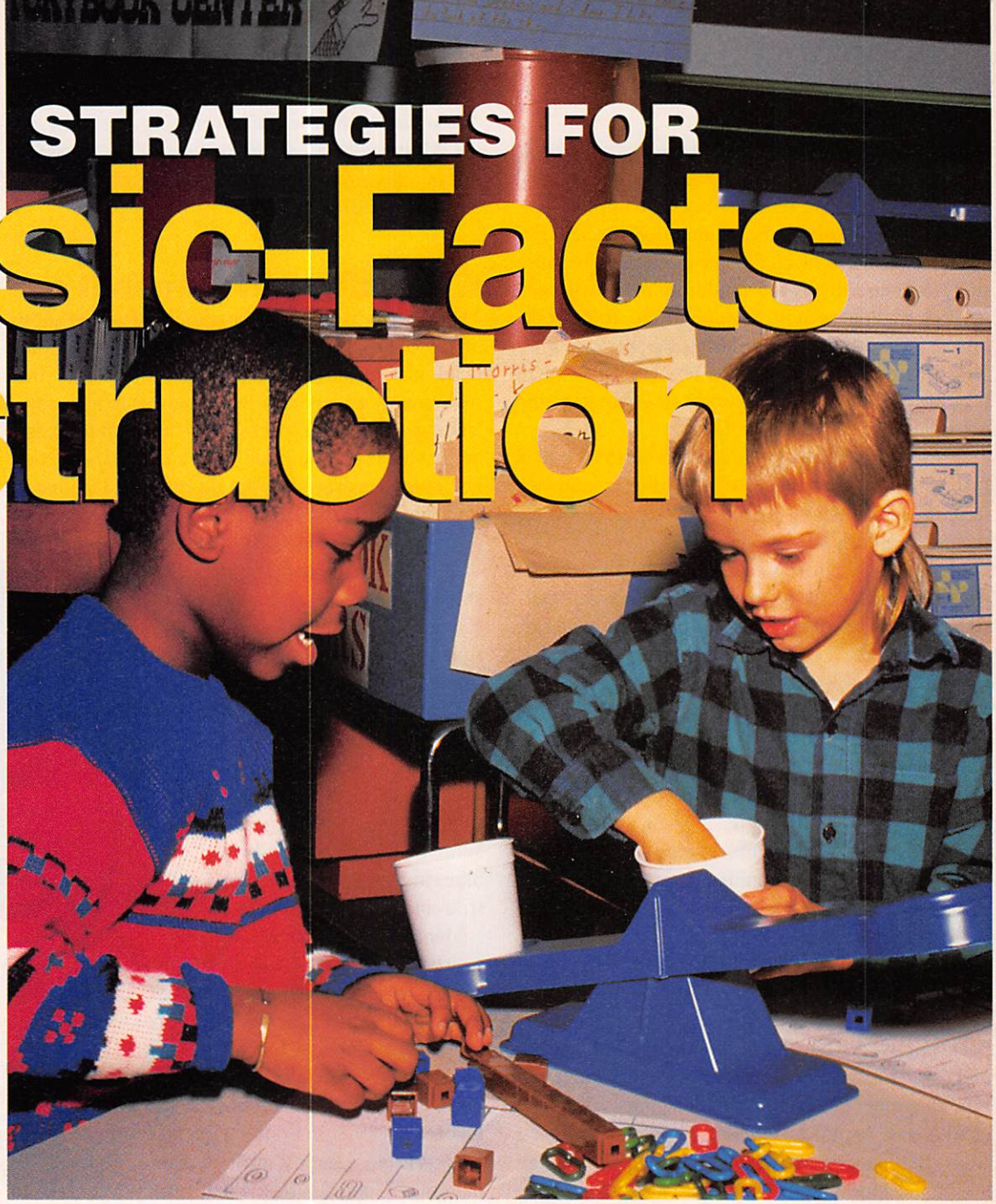
The mathematics classroom is a place for thinking and learning. Such activities as these actively involve students in thinking carefully about attributes. As they identify attributes, they begin to see patterns. Opportunities exist for connections to geometry, number and operation sense, and logic and reasoning. But the students do not always see the educational opportunities inherent in such activities. The students simply think that the activities are a lot of fun.

Reference

Bird, Elliott. "Counting A-Blocks: Constructing Meaning for the Multiplication Principle." *Mathematics Teaching in the Middle School,* in press.

For more ideas on further explorations, see the K–6 teacher's guides of the Comprehensive School Mathematics Program, published by the Mid-Continent Regional Educational Laboratory (McREL). The attribute block set is also available from McREL as either the A-Block String Game Kit (item 17-8910, $5) or the A-Block Game Minipackage (item 17-8969, $10). Contact McREL-CSMP, 2550 South Parker Road, Suite 500, Aurora, CO 80014. ▲

STRATEGIES FOR Basic-Facts Instruction



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Andrew C. Isaacs and William M. Carroll

In a class of beginning second graders, children explained with these typical replies how they solved the number fact 8 + 7: "I know 7 + 7 is 14, and 1 more is 15"; "8 + 2 makes 10. But 7 has 5 more, so the answer is 15"; and "I just knew the answer was 15." Teaching basic number facts like 8 + 7 has been a goal of elementary mathematics instruction for more than 100 years and continues to be important today.

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Although most teachers agree that students' fact mastery is important, many are unclear about how to seek it in ways that are consistent with the NCTM's Standards (1989, 1991, 1995). They even disagree about what knowing the basic facts means and when, or even if, students should achieve mastery. Is it appropriate to expect first graders to memorize addition facts, or will this task interfere with their mathematical thinking? What classroom practices can build both understanding and quick recall? Can fact mastery be achieved through problem-solving activities, or is practice necessary? If current reforms in mathematics education are to succeed, questions about the basic facts need answers.

Why Should Children Learn the Facts?

Most people recognize that children should learn the basic facts because knowing them is useful, both in school and in life out of school. Estimation and mental computation require the use of basic facts. How can students use 80×40 to estimate 84×41 if they do not know 8×4? Students who know their facts do better in school mathematics. Parents, teachers, and the public expect schools to teach the basic facts.

Fortunately, no conflict need exist between fact mastery and school mathematics reform. Many goals of reform—helping students make connections between school mathematics and the real world, helping students develop conceptual understanding as well as procedural skills, helping students learn to explain their thinking and to understand others' explanations—can be achieved through a program that also leads to fact mastery. Properly approached, the basic facts offer excellent opportunities for teaching children to think mathematically.

How Should the Facts Be Taught?

The traditional rote approach to the basic facts, with frequent drill and timed tests, has serious disadvantages. Premature demands for quick performance can induce anxiety and undermine understanding. Rigid schedules for mastery do not accommodate individual differences and have unfortunate outcomes for some children. The rote approach encourages students to believe that mathematics is more memorizing than thinking.

Today, however, the outlines of a better approach are clear. This approach begins with children's natural thinking. The essence of many current reforms in primary-grade mathematics, including the approach to the basic facts described here, is to recognize and build on the wealth of informal mathematical knowledge that children bring to school. Traditionally, much of this knowledge has been ignored or suppressed.

Early work with basic facts should help children refine and extend their natural strategies for solving simple problems. As children increase their proficiency at various strategies, they begin to remember the simplest facts. Knowing the simpler facts makes possible more efficient strategies for harder facts. Gradually, students master more and more efficient strategies and commit more and more facts to memory. At the end of the process, students can accurately and automatically produce all the basic number combinations. Many of these combinations are recalled from memory, but a few may also be found through quickly executed strategies or suitable rules. Fact strategies and recall are used by both children and adults. We know a research mathematician, for example, who solves 8×7 by doubling 7 three times—14, 28, 56—but he does the doubling so quickly and effortlessly that it is automatic.

In this section of the article, we describe this "strategies" approach for the addition and subtraction facts; many of the same ideas also apply to the multiplication and division facts. First we describe how children's informal knowledge, especially their knowledge of counting and of part-whole relationships, can be used in beginning fact work. Then we describe how children use facts that they know to derive facts that they do not know. Finally, we discuss the role of practice and sketch a possible sequence for addition- and subtraction-facts instruction.

No conflict need exist between fact mastery and mathematics reform

Counting to solve problems

Perhaps the best way to extend primary-grade children's informal understanding of addition and subtraction is by asking them to solve simple problems—without telling them how those problems are to be solved. These problems can come from real life, classroom situations, textbooks, the teacher, or the children's imaginations: "An adult movie ticket is \$7 and a child's ticket is \$4. How much for one adult and one child?" "Miriam wants a toy horse that costs \$15. She has \$8. How much more does she need?"

As students encounter the problems, they should be encouraged to devise their own solution procedures by looking for patterns, thinking logically, and using manipulatives. The adult approach—reducing such problems to addition or subtraction number sentences and retrieving the answers from memory—is not a natural strategy for young children. Instead, primary-grade children tend to use direct modeling, counting, and derived-fact strategies (Bergeron and Herscovics 1990).

Direct-modeling techniques are generally the first to appear for a given type of problem. The child counts out objects to represent the quantities in a problem, performs actions with the objects that parallel the problem situation, and counts some set to find the answer. To solve the preceding movie-ticket problem, for example, a child might count out seven chips for the cost of the adult's ticket and four chips for the child's ticket. Then by counting all the chips, the child can find the total cost. Direct modeling is used by young students to solve simple addition and subtraction problems and even some

surprisingly difficult multiplication and division problems (Carpenter et al. 1993).

Direct modeling can be rather inefficient, however, especially for problems with larger numbers. Eventually, direct-modeling strategies are supplanted by oral or mental counting strategies. A large number of such strategies for addition have been identified (Resnick 1983; Carpenter and Moser 1984; Baroody and Ginsburg 1986; Siegler and Jenkins 1989). Two common strategies are "counting all" and "counting on from the larger addend."

A significant feature of most counting strategies is that the child must keep track of how many numbers have been said. To solve 3 + 4 by counting all, the child first counts three numbers (1, 2, 3) and then four more numbers (4, 5, 6, 7). This "double counting" can be tricky; using fingers or objects can help. Similar counting strategies exist for subtraction, such as counting up from the smaller number to the larger and counting back from the larger number.

How best to help children advance to more efficient strategies is an open question. Certainly laying out a strict sequence of strategies and expecting all children to adhere to it would be ill-advised. Not only do different children progress at different rates, but the same child may use different strategies on different problems or even on the same problem in different contexts. However, if teachers hesitate too much to demonstrate better methods, students' progress may be impeded.

One approach to encourage more efficient methods is to ask children to share their strategies. This method helps them improve their communication skills and learn from one another. **Figure 1,** for example, shows how two first graders used a hundreds chart to find 5 + 9. In a typical class, children will use and describe various approaches, so most children will encounter new but understandable techniques. The teacher may also propose and model strategies, taking care that certain strategies do not become "official" while other strategies are discouraged. The teacher should not be disappointed when a child does not adopt more efficient strategies right away—development may be advancing below the surface at the rate best suited to the child.

To encourage more efficient methods, ask children to share strategies

Class discussion of strategies should be supplemented with exercises designed to facilitate more sophisticated strategies. For example, the advance from counting all to counting on depends in part on skill in counting ahead the correct number of counts from another number. By practicing counting outside any problem context, children can develop competencies that support more sophisticated problem-solving strategies. A variety of such exercises should be included: counting forward and backward by 1's, starting at various numbers; skip counting, especially by 2's, 5's, and 10's; and counting forward and backward a given number of numbers: "Start at 8 and count forward 3" or "Start at 11 and count back 2," and so on.

Parts and wholes

Another central understanding that young children bring to school is that a quantity can be broken into parts that taken together equal the original quantity. They also understand that if they have some and get more, then they end up with more; and if they have some and lose some, then they end up with less (Resnick, Lesgold, and Bill 1990). Developing these basic "parts and whole" ideas further is essential to understanding addition and subtraction.

Ten-frames, like those in **figure 2,** are good for developing part-whole understandings involving the landmark numbers 5 and 10 (Thompson and Van de Walle 1984; Thornton and Smith 1988; Van de Walle 1994). These understandings are especially useful in addition- and subtraction-fact work. For example, the ten-frame for 8 in **figure 2** shows that 8 is 3 more than 5 and also 2 less than 10. The ten-frame for 4 shows that 4 is 1 less than 5 and 6 less than 10. Once students learn facts involving 5 and 10, especially the pairs of numbers that sum to 10, they can use their knowledge to solve other basic-fact problems.

Derived facts

Although most young children do not have automatic command of the basic facts, most adults do. In between is a stage in which some facts are known and others are not. During this stage, many children use the facts that they know to derive the facts that they do not know. Class discussion of such derived-fact strategies helps students learn from their peers and also legitimizes the use of strategies, thus encouraging the invention of further strategies (Steinberg 1985). Class discussion should examine the relative advantages of different strategies for various problems (Thornton and Smith 1988). Encouraging the discussion of multiple solutions enhances strategy and fact knowledge and helps students develop methods for mental and multidigit computation. Instruction to facilitate specific strategies can also be worthwhile.

The "doubles" facts are often useful for deriving unknown facts. For example, a child might solve 3 + 4 by noting that 3 + 3 = 6, so 3 + 4 must be 1 more than 6. Facts like 8 + 6 can be solved either by "sharing" (8 + 6 = 7 + 7 = 14) or by using a double and

adding 2 more (8 + 6 = 6 + 6 + 2 = 12 + 2 = 14). Since doubles-based strategies are common, care should be taken that children learn the doubles facts early. Many games can be modified so that they involve doubles. For example, games with two dice can be played with one die doubled instead. A chart with examples of addition doubles, such as 6 eggs + 6 eggs = 12 eggs, can be kept as a class project and explored for patterns, for example, all the sums are even. Brief oral drills are also appropriate as children are consolidating their knowledge of these facts.

Many other common strategies involve 10. For example, a child using 10 might solve 9 + 7 in several steps: 9 + 1 = 10 and 10 + 6 = 16, so 9 + (1 + 6) = 16. To support such strategies, early attention should be given to complements of 10, such as 6 + 4, 7 + 3, and so on. The ten-frame activities described previously are ideal.

Children also use derived-fact strategies for subtraction. Some subtraction strategies are refinements in counting, such as using 10 as a bridge in counting up or down. For example, to solve 13 – 6, count up 4 from 6 to 10, and then up 3 more from 10 to 13, for a total counted of 4 + 3 = 7. Other strategies involve using known addition facts to derive unknown subtraction facts: 15 – 8 = 7, since 7 + 8 = 15.

Practice

The place of practice in school mathematics is much disputed. We think that a reasonable position was described by William Brownell more than fifty years ago. Brownell and his student Charlotte Chazal found that under certain conditions, practice can be harmful. Premature demands for speed, for example, caused many children simply to become quicker at immature approaches. Delaying drill was found to result in better understanding and ultimately in less need for drill (Brownell and Chazal 1935). Over the years, unfortunately, some educators have misunderstood this and similar research and have concluded that all practice is bad.

We believe that the right conclusion is that premature practice can be detrimental but that properly managed practice is essential in the development of expertise—whether the subject is piano, tennis, or the basic facts (Brownell 1956; Chase and Chi 1981; Siegler 1988; Anderson, Reder, and Simon 1996). Brief, engaging, and purposeful practice distributed over time is usually most effective. Problem solving is one important source for such practice, but games, computers, or even old-fashioned technology like flash cards and choral drills can also be useful.

An instructional sequence

The preceding ideas can be used to sketch a possible instructional sequence for the addition and subtraction facts. Note that in this sequence, the facts are grouped by strategy rather than by sum. A double like 6 + 6, for example, may be easier than a problem like 4 + 3 and, accordingly, appears earlier in this sequence.

1. Basic concepts of addition; direct modeling and "counting all" for addition
2. The 0 and 1 addition facts; "counting on"; adding 2
3. Doubles (6 + 6, 8 + 8, etc.)

FIGURE 1

Hundred-chart strategies for 9 + 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

"I started at 5. Then I counted 5 more to 10, then 4 more to 14."

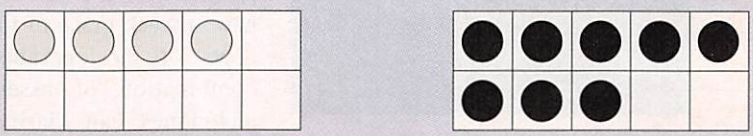
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

"I started at 5 and jumped ahead 10 to 15. But it was only 9, so I moved back 1 to 14."

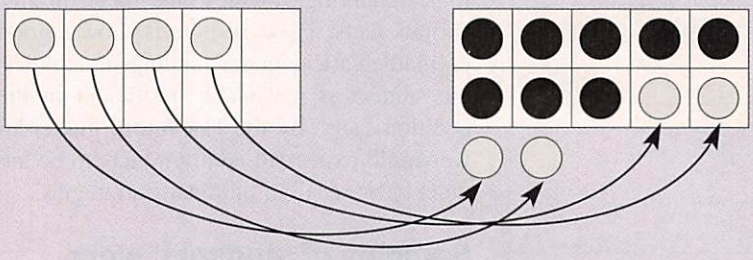
FIGURE 2

Ten-frames showing that 4 + 8 = 12

4 + 8 = _____



Maria made 4 and 8 on the ten-frames. Then she moved 2 counters to make 10 and 2 more to make 12. So 4 + 8 = 12.



4. Complements of 10 (9 + 1, 8 + 2, etc.)
5. Basic concepts of subtraction; direct modeling for subtraction
6. Easy subtraction facts (– 0, – 1, and – 2 facts); "counting back" to subtract
7. Harder addition facts; derived-fact strategies for addition (near doubles, over-10 facts)
8. "Counting up" to subtract
9. Harder subtraction facts; derived-fact strategies for subtraction (using addition facts, over-10 facts)

How Can Fact Knowledge Be Assessed?

The assessment of children's fact knowledge should be balanced, based on multiple indicators, and aligned with instruction. Assessment should help the teacher evaluate not only answers but also how students are getting those answers and whether students understand the underlying mathematical concepts and connections. For example, a student might appear to know the basic facts during problem-solving activities but actually be relying on counting. Another student might be quite proficient on isolated facts but have a weak grasp of the concepts of the operations. A combination of assessment techniques can clarify each student's strengths and weaknesses and can help the teacher plan instruction.

What is meant by fact proficiency differs by age

What is meant by fact proficiency differs by age. For example, the first grader in **figure 3** used finger counting, doubles, and recall in answering various facts. These responses show a good range of mathematical understanding and indicate that the student is reasonably proficient in the basic addition facts. By third or fourth grade, however, we would expect all addition facts to be answered quickly by recall or automatic strategies.

Samples of students' work

Collecting samples of student work is a good way to gather evidence about students' knowledge and application of facts. These performance-based samples should come from activities in which students use facts. For example, **figure 4** shows a number-collection box in which a second grader recorded different ways of making a target number, in this example, 9. Such exercises help children develop their understanding of addition and subtraction and also afford opportunities for assessing fact knowledge.

Although performance-based samples offer evidence of conceptual understanding and applications, information about students' level of proficiency is often limited. Typically, for example, work samples do not reveal whether the student used counting, derived-fact strategies, or recall. The information is also limited to the particular numbers involved in the sample. Without additional information, a teacher might find it difficult to plan meaningful instruction.

Observations, class discussion, and interviews

Observing students engaged in games and problem-solving activities can yield rich information about their fact knowledge. For example, as students play a game, a teacher may notice whether they are using counting strategies, derived-fact strategies, or known facts. More important, the teacher can get a better idea of the range of students' knowledge with individual facts. Brief observational notes can help with planning individual and whole-class instruction: Tomás is still counting on, even for the easy facts; Juanita knows most of the double facts, and she also uses these facts to solve some of the near doubles. Useful information can also be obtained during class discussions as individuals explain their solutions to story problems or other problem-solving activities.

Short individual interviews are probably the best way to get a full picture of a student's progress with basic facts. Although these interviews are time-consuming, with a little planning a teacher can manage a five-minute interview twice a year with each student, perhaps spacing the interviews over a month. More frequent interviews with students who are having difficulties can help pinpoint problems.

Inventory tests

Although clearly an overreliance on timed tests is more harmful than beneficial (Burns 1995), this fact has sometimes been misinterpreted as meaning that they should never be used. On the contrary, if we wish to assess fact proficiency, time is important. Timed tests also serve the important purpose of communicating to students and parents that basic-fact proficiency is an explicit goal of the mathematics program. However, daily, or even weekly or monthly, timed tests are unnecessary.

An inventory test on all the addition and subtraction facts might be done at the beginning of second and third grades. These tests establish a baseline for measuring progress and provide information that can be useful in planning instruction. End-of-the-year tests, and perhaps mid-year tests, can be used to document progress. Similar inventories for multiplication and division facts might be given in fourth and fifth grades. We recommend

FIGURE 3

A "reasonably proficient" first-grade student

Student: [Student reads] Three plus five equals . . . . Hm. [Pause] Three. [Student then counts on fingers, putting up five fingers at one time.] Four, five six, seven, eight. Eight.
Teacher: How did you figure that out?
Student: I did it on my fingers.
[Child is shown card with 5 + 5 on it.]
Student: [Rapidly] Five plus five equals ten.
Teacher: How did you get that?
Student: I figured it out in my mind.
Teacher: You always knew it? [Student indicates yes.] Okay, what's six plus six?
Student: [Fairly rapidly] Thirteen.
Teacher: How did you get that?
Student: Because I counted five and then added two more: five plus five and two more.
Teacher: And you got what, thirteen?
Student: Yeah. [Child reads next problem] "Seven plus nine equals. . . . " [Pause. Then child begins to count on fingers. First, child apparently begins to count all—to seven on one hand. Then starts over, saying seven and starting over on fingers, putting up nine fingers one at a time.] Eight, nine, ten, . . . , sixteen. Sixteen.
Teacher: Sixteen. Okay, so here's another one. If seven plus nine is sixteen, what's nine plus seven?
Student: [Two seconds, then child responds with enthusiasm.] Sixteen!
Teacher: How do you know that?
Student: It doesn't matter which one's first. But they're always, . . . they're just always like, . . . no matter what is first, they're always the same number.
Teacher: Here's another one. Four plus ten.
Student: [Quickly] Four plus ten is fourteen.
Teacher: How did you get that?
Student: Well, I just figured it out.
Teacher: On your fingers.
Student: No. I thought.

against any timed tests during first grade, or any frequent use in the primary grades, because they work against a strategies approach to the facts. That is, in a timed situation, students will be less likely to explore the more sophisticated strategies necessary to make progress.

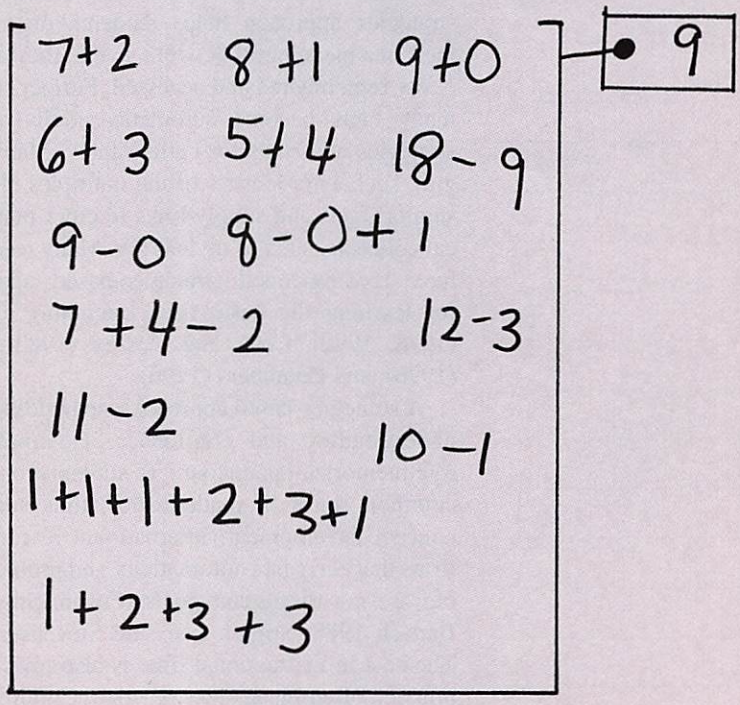
Small-scale diagnostic tests

Although positive inventory-test results are reassuring, they yield limited information. It may be, for example, that a student is proficient at some facts but uses counting for other facts. For this reason, it is helpful to test smaller sets of facts with short diagnostic tests linked to specific strategies. For example, after students work on the doubles, a quick test of the doubles facts can indicate whether students are ready to move on. As students move toward proficiency, short tests of mixed fact strategies—doubles, near doubles, and complements of 10—can also be useful for diagnostic purposes. A three-second rule is often used as a benchmark of automaticity (Van de Walle 1994), although some teachers prefer two seconds (Thornton 1990). Note that these criteria allow enough time for students to use efficient strategies or rules for some facts.

If we expect students to move from counting

FIGURE 4

"Number-collection box" for 9



strategies toward facility with facts, then an occasional low-stress test or practice is consistent with our goals and the message that we want students to receive. The crucial point is to emphasize individual progress. In kindergarten and first grade, counting strategies are appropriate for solving the basic addition facts, but we should have concerns about students who are still "counting all" in the middle of second grade or who have not mastered even the easiest addition facts. Not diagnosing these students' difficulties and planning appropriate instruction for them does them a disservice. A balanced approach to assessment—work samples, some observations, some test information, and some interview information—gives the teacher, the student, and the parent a more complete portrait of the child's fact knowledge, how it is connected to other mathematical knowledge, and what progress is being made.

Neglecting basic facts may undermine reforms now under way

Conclusion

A strategies-based approach to the basic facts has several advantages. First of all, it works: children do learn their facts. Rathmell (1978) found that teaching children thinking strategies facilitates their learning and retention of basic facts. More recent studies have confirmed this effect again and again. These findings should not be surprising: a strategies approach helps students organize the facts in a meaningful network so that they are more easily remembered and accessed. Further, although many facts become automatic, adults also use strategies and rules for certain facts. Many strategies, such as properties of the multiples of 9, both support facts and supply links to other mathematical concepts, such as divisibility. Many researchers have recommended strategies-based approaches for learning the basic facts, including Thornton (1978, 1990), Cook and Dossey (1982), Myren (1996), and Chambers (1996).

A strategies-based approach also builds students' understanding and confidence. De-emphasizing rote memorization encourages students to use their common sense in mathematics, thus supporting concept development. International research confirms that early fact automaticity and problem solving are not discrepant goals (Fuson, Stigler, and Bartsch 1988; Stigler, Lee, and Stevenson 1990). The cost in instructional time is also low: delayed practice often means less practice. Children's success at learning their facts also reassures parents about their children's mathematics program.

Certain pitfalls must be avoided in a strategies-based approach. One danger is that children might learn strategies by rote, so that mindless memorization is replaced by equally mindless "strategies" (Cobb 1985). Another possibility is that class discussion might degenerate into the tedious recitation of every imaginable method, with little critical appraisal of the various approaches. Encouraging multiple ways to solve fact problems may also lead students to conclude that memorizing the facts is not important. We believe, however, that in most situations a thoughtful and sensitive teacher can avoid these hazards.

Our purpose has been to address important questions about the basic facts, for fear that neglecting them will undermine reforms now under way. We worry that our efforts to correct for a narrow focus on lower-level skills will lead to an overcorrection. We recall Brownell's warning at the beginning of the New Math era: "In objecting to the emphasis on drill prevalent not so long ago, we may have failed to point out that practice for proficiency in skills has its place too" (1956). We must remember that successful education involves both basic skills and higher-order processes.

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USING ALTERNATIVE ALGORITHMS WITH Preservice

During their undergraduate program, preservice elementary teachers are expected not only to become generalists across a wide range of school subjects but also to develop pedagogical knowledge of the developmental and social needs of children. We have developed a two-semester freshman-level mathematics-content course that attempts to address multiple needs of preservice teachers. The goals of this course are to help preservice teachers *(a)* deepen their understanding of mathematical concepts; *(b)* restructure their attitudes toward, and beliefs about, the nature of mathematics and how it is learned; *(c)* investigate pedagogical issues; and *(d)* experience mathematics learning within a reform-based environment. These goals are interrelated and reflect the complexity of the nature of the knowledge and disposition, both mathematical and pedagogical, that preservice teachers are expected to develop.

Linda M. Simonsen and Anne R. Teppo

The NCTM's *Assessment Standards for School Mathematics* (1995) defines mathematical disposition as "interest in, and appreciation for, mathematics; a tendency to think and act in positive ways; includes confidence, curiosity, perseverance, flexibility, inventiveness, and reflectivity in doing mathematics" (p. 88). We define what we call *pedagogical disposition* related to the teaching of mathematics as the ability to understand the explanations of others (Morse and Wagner 1998), the tendency to search for and appreciate multiple solution strategies, and the inclination to recognize the value of collaborative effort. This article presents a set of tasks using a child's invented strategy and a nonstandard algorithm to illustrate one way to facilitate the development of both mathematical and pedagogical dispositions.

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Children's Invented Strategies

Our course used an example of a child's invented strategy for multidigit subtraction and a pair of nonstandard algorithms for multiplication to cause preservice teachers to question their beliefs about the role of standard algorithms in elementary school mathematics. The activities required them to think in different ways about the mathematics that many of them had accepted as rule bound and mysterious and helped them discover the mathematical structure embedded in the computational algorithms.

On the second day of the semester, the students were presented with the strategy called "Stephen's running total," which was invented by a group of elementary children (see **fig. 1**) (Gravemeijer, van den Heuvel, and Streefland 1990, 43). While working in small groups, the students were asked to explain why Stephen's strategy produced the correct answer and to use this algorithm to solve a similar multidigit-subtraction problem. The class discussion that followed helped in assessing their abilities to uncover the mathematics embedded in the algorithm and revealed a wide range of beliefs about school mathematics.

Teachers

The steps that constitute Stephen's strategy illustrate the way that children, if not taught otherwise, invariably proceed from left to right when adding or subtracting (Kamii, Lewis, and Livingston 1993). The numbers that Stephen uses in each step preserve place value. For example, in the first step, the child subtracts 3000 from 8000 rather than 3 from 8. In the second step, the child subtracts a larger number from a smaller number by working with a deficit. In the third step, the child deals with the deficit by removing 400 from the running total of 5000. By working in this way, the child shows strong number sense throughout the calculation.

Many preservice teachers in our course had experienced traditional school mathematics programs. In their own minds, mathematics largely consisted of meaningless memorization of basic facts, rules, and procedures. From past experience, they perceived their future roles to include teaching children sets of standard computational procedures.

The influence of these traditional backgrounds was evident in the students' contributions to the class discussion of Stephen's running total. Many reported feeling frustrated trying to understand a method that was different from the familiar standard algorithm for subtraction. Their conceptions of multidigit calculations were constrained by the way in which the standard algorithm treats all digits in the problem as numbers less than 10. The concept of place value is not explicit in the rules of computation that they had learned. One teacher commented that the process of "crossing out the 7, changing it to a 6, and adding the 1 to the other 1" had always confused her as a child. She could never understand why "adding the 1 produced 11 rather than 2."

The students expressed doubt that children could understand Stephen's method. Many students thought that it would be better to teach children the standard algorithm. This discussion revealed their belief that teachers must "teach" a method that they had learned themselves; if teachers were familiar with the method, then it would be acceptable to *their* students. Even though the preservice teachers were bewildered when asked to perform computations by an unfamiliar method, they were not able to transfer this experience to that of their future students.

FIGURE 1

Stephen's running total

8371	8000	− 3000	=	5000
−3754	700	− 300	=	400
	5000	− 400	=	4600
	70	− 50	=	20
	4600	+ 20	=	4620
	4620	− 3	=	4617

How can you explain the combined usage of addition and subtraction in Stephen's method?

One purpose of this sequence of activities was to help make the preservice teachers' beliefs explicit. The attitudes expressed at the beginning of the semester were to be expected from individuals who had experienced a more rule-based school mathematics experience. A goal of the course was to offer activities that would not only challenge these beliefs but help develop alternative perspectives. By modeling addition and subtraction problems using manipulatives and studying a child's invented strategy for subtraction, the students in our course were exposed to a vision of school mathematics that encouraged different ways of thinking about and solving problems. Asking our students to think in a new way about arithmetic calculations that had become routine to them helped them reveal the underlying mathematical structure.

Alternative Algorithms

During the third week of the semester, we presented to the students a set of alternative multiplication algorithms, one of which was the "Russian peasant" method **(fig. 2)** (Musser and Burger 1997, 164). They tested their understanding of the steps in each algorithm by applying the rules to calculate answers to additional problems. In the subsequent class discussion, we focused on analyzing the mathematics embedded in the procedural steps. (See **fig. 3** for an explanation of the Russian-peasant method.)

FIGURE 2

The Russian-peasant method for multiplication

The Russian-peasant algorithm for multiplying 27×51 is illustrated as follows:

Halving		Doubling
27	×	51
13	×	102
~~6~~	~~×~~	~~204~~
3	×	408
1	×	816

Notice that the numbers in the first column are halved, disregarding any remainder, and the numbers in the second column are doubled. When 1 is reached in the halving column, the process is stopped. Next each row with an even number in the halving column is crossed out and the remaining numbers in the doubling column are added. Thus,

$$27 \times 51 = 51 + 102 + 408 + 816 = 1377.$$

FIGURE 3

An explanation of the Russian-peasant method for multiplication

The process of halving the numbers in the first column and doubling the numbers in the second column preserves the product of the multiplication. However, the algorithm appears to disregard the remainders when an odd number is halved. Using the distributive property, the following calculations show the remainders, thus demonstrating how the algorithm works.

$$27 \times 51 =$$

$$\left(\frac{1}{2}\right)(26+1) \times 2(51) =$$

$$\left(\frac{1}{2}\right)(26) \times 2(51) + \left(\frac{1}{2}\right)(1) \times 2(51) =$$

$$13 \times 102 + 51 =$$

$$\left(\frac{1}{2}\right)(12+1) \times 2(102) + 51 =$$

$$\left(\frac{1}{2}\right)(12) \times 2(102) + \left(\frac{1}{2}\right)(1) \times 2(102) + 51 =$$

$$6 \times 204 + 102 + 51 =$$

$$\left(\frac{1}{2}\right)(6) \times 2(204) + 102 + 51 =$$

$$3 \times 408 + 102 + 51 =$$

$$\left(\frac{1}{2}\right)(2+1) \times 2(408) + 102 + 51 =$$

$$\left(\frac{1}{2}\right)(2) \times 2(408) + \left(\frac{1}{2}\right)(1) \times 2(408) + 102 + 51 =$$

$$1 \times 816 + 408 + 102 + 51 = 1377$$

Although the preservice teachers were able to use the algorithms, few were able to explain satisfactorily the mathematics behind the Russian-peasant method. The instructor provided key information about the use of remainders to help the class make sense of the sequence of steps. As with Stephen's running total, the discussion was a way to assess their understanding of the underlying mathematical structure and their attitudes toward the exercise.

We have found that presenting alternative algorithms in problem-solving situations helps our students deepen their understanding of the mathematics behind computational procedures. The problems are challenging in that solving them requires students to examine familiar mathematics in new ways. Thus, these problems are accessible by, and of interest to, mathematically weak as well as strong students. The problems are satisfying because their solutions lead to deeper conceptual understandings and help demystify computational procedures.

By studying these nonstandard algorithms, preservice teachers become aware of mathematics as a human discipline. Many are surprised that alternative "rules" exist to the standard paper-and-pencil procedures that they were taught; see Philipp (1996) for a discussion of the use of multiculturally based algorithms in this context. These future teachers begin to recognize that the ability to understand and accept as legitimate mathematics different, nonstandard sets of procedures is a valuable skill.

The following comments about the Russian-peasant method were written by our students as part of their midsemester portfolio reflections. Their comments reveal the growing importance that they were attaching to *understanding* the mathematics they were using. In addition, they were beginning to perceive the value in working through frustrations to reach a solution collaboratively.

> I chose this problem because it is such a simple-looking algorithm, and yet it has such complex dynamics. The process one must go through to complete the correct answer is simple in concept, but to explain why it works is very difficult. The problem taught me that understanding is more important than doing. Understanding concepts is where the real challenge and full understanding [lie].

[This problem was not] hard to figure out, but I was impressed at how everybody in the class suddenly became interested in figuring out and understanding how it worked. The entire class started working with each other and collaborating with each other. It seemed that nobody was afraid to voice [his or her] opinions. I also think that because everybody worked so diligently on it, nobody has forgotten why it works or for that matter ever will.

The Russian-peasant method really opened my eyes. I thought this was one of those magic math problems. It worked, but there seemed to be some trick involved. I didn't completely understand the problem until somebody said there were remainders. "Remainders" made the whole problem make sense to me. It took the mystery out of math for me.

This problem was probably the first problem in class that had me figuring and testing over and *over* to see if it always worked. It did! I enjoyed finding out a new algorithm. This problem opened my eyes to see that there are many different ways to solve mathematics problems.

Conclusions

We have found that alternative algorithms and children's invented strategies can be used effectively in a mathematics-content course. In such a course, preservice elementary teachers can enhance their mathematical and pedagogical dispositions. Activities using these algorithms help students with a range of mathematical abilities expand their perceptions of mathematics from that of a rule-based and single-answer discipline to one that involves of multiple approaches and personal constructs. The exercise of trying to understand and communicate about an alternative set of algorithms gives preservice teachers valuable experience in seeing mathematics from different perspectives. This skill will serve them well in encouraging their future students to construct their own mathematics.

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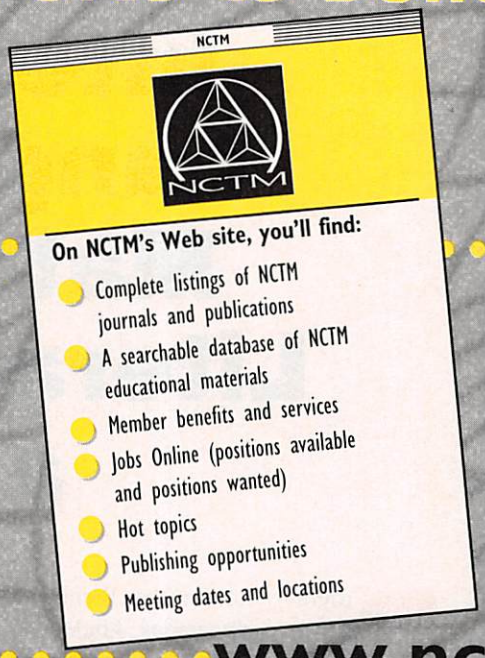

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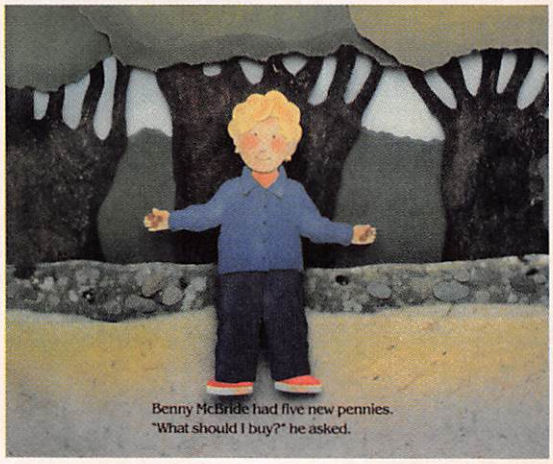
Links to Literature

Interweaving Language and Mathematics Literacy through a Story

Jacqueline Harris

Programs that promote literacy development in the early grades, kindergarten through grade 2, are of prime interest to educators, particularly when they are also effective for ESL students. With interest, then, I talked with Silvana Bozzato, a kindergarten teacher at Our Lady of Victory School, set in a multicultural, downtown Toronto neighborhood. I wanted to know if she used in her program any books with a focus on mathematics, wondering how that practice might help students understand important mathematical concepts. Bozzato used many such books, and we agreed that she would model one lesson in which mathematics and language were interwoven, choosing a beautifully illustrated, lyrical story by Pat Brisson, *Benny's Pennies* (1993).



Illustrations by Bob Barner, © 1993 by Bob Barner, from *Benny's Pennies* by Pat Brisson. Used by permission of Random House Children's Books, a division of Random House, Inc. All rights reserved.

Making the Mathematics Connection

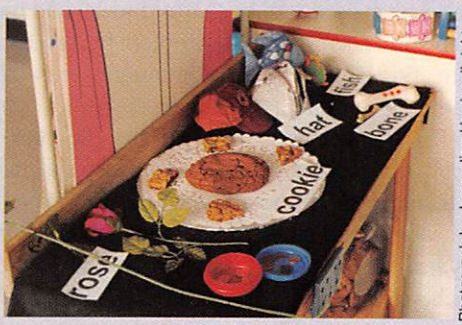
Benny's Pennies is a story about a young boy who has five pennies to spend. He buys for one penny each a rose, a cookie, a paper hat, a bone, and a fish. When he arrives back home, he gives his purchases to his family. His mother receives the rose; his brother, the cookie; his sister, the hat; the family cat, the fish; and the family dog, the bone.

With literacy and mathematical outcomes in mind, Bozzato carefully planned her introduction to the book. She gathered props based on the story, such as roses, cookies, hats, bones, fish, and pennies, but the props had numerous variations. She chose roses of several sizes but of the same color. The cookies—chocolate chip—differed in size and shape. Hats of different sizes, shapes, and materials; two fish, one plain and one with rainbow-colored scales; and two bones of different sizes, one with three spots, completed the list. Two containers each held five pennies. In addition, Bozzato had prepared labels with the name of each prop: cookie, hat, rose, bone, fish, and penny (see **fig. 1**).

Prepared by the editor, **Jacqueline Harris,** *York University Faculty of Education, 70 Bainbridge Avenue, North York, ON M3H 2K2. Readers are encouraged to share experiences using children's books with* Teaching Children Mathematics. *Please send such descriptions to the editor.*

FIGURE 1

The props of the story



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The Lesson: Questioning and Scaffolding Ideas

Bozzato placed the props on a table with a window backdrop (see **fig. 1**) and asked small groups of students to examine the items. The students, familiar with this type of activity, went silently to look at the props, occasionally whispering and pointing out interesting details and characteristics of some of the objects and anticipating questions that the teacher might ask in a whole-class discussion. After all students examined the items, Bozzato began questioning the class, starting with their recall of details.

Teacher: I want you to close your eyes for a minute and try to remember . . . some or all of the things you saw on the table . . . words or objects. . . .

Shanice: Cookies.

Teacher: Do you remember how many cookies?

Shanice: More than one.

Alessia: Five.

Teacher: Five! Good for you. Are the cookies all the same size?

The class: No.

Teacher: No? What sizes did you see?

Christopher: Big, short, small, medium, and large.

Teacher: Did you notice the kind of cookies?

Timothy: Chocolate chip.

Teacher: Did you notice anything else . . . the way the cookies are made?

Sheldon: Shapes.

Teacher: Shapes! Good for Sheldon. He saw shapes. What was the shape of the biggest one? The smallest? The others?

The questioning continued, with Bozzato encouraging the students to use their memories and visual cues to recall and compare the items, distinguishing by such characteristics as size, color, shape, and material. She also asked them to solve problems by estimating size. "Show me with your hands how long you think the stem is. . . . Now show me how small you think the smaller one is—with your fingers."

The questions gave the students many opportunities to familiarize themselves, using their oral and visual skills, with the vocabulary in the upcoming story. Specifically, they learned the descriptive and detailed words related to the characteristics of objects, many of which are terms that are essential for the mathematical understanding of numeration, for example, *how much, how many, difference;* measurement, for example, *estimate, how long, size;* geometry, for example, the names of two-dimensional shapes; and patterning and algebra, for example, comparing and contrasting items. The questioning time, which included both closed and open-ended questions, allowed students to describe particulars, make clarifications, articulate their ideas, and use their problem-solving skills. This "scaffolding through questions" is a central part of literacy development (Heath 1986, 166), as well as an important component of one standard developed by the National Council of Teachers of Mathematics (1989), Mathematics as Communication.

The questioning period had been lively and anticipatory of the story to come. Bozzato next refocused the group by passing around the two containers holding the pennies, asking them to look at the items (see **fig. 2**). She asked, "What's in the container?" The students answered, "Money." She continued, "What kind of money?" The class answered, "Pennies," and she asked again, "How many pennies?" Jordan responded, "Five." "Is Jordan right?" she asked, and the class answered, "Yes."

FIGURE 2

Students examine the pennies.

Photograph by Jacqueline Harris; all rights reserved

Reading and Recounting the Story: Interweaving Mathematics and Language

Bozzato made a transition from the questioning to the book by telling the students that she was going to read a story that would include all the items on the table. Next, she asked more questions, this time about the book itself, encouraging students to predict the story from the picture on the cover, to sound out the words, and to identify the rhyme in the title (see **fig. 3**).

As she began the story, Bozzato ensured that the students were following it by asking a few direct questions; for example, after reading the first page, she asked, "How many pennies? New or old? How many in one hand? The other? How many all together?" The entire class quickly counted, and the students held up their fingers to demonstrate their answers to one another. The teacher surveyed the finger responses for accuracy before continuing.

The next illustration was of Benny's family. Bozzato had another mathematical opportunity: "How many pets and people?" "Two pets and three people," Said Tamara. "Five," said the class. But one student said, "Four," so Bozzato stopped. "Let's try that again," she suggested. "Let's count the people first—one, two, three—and the pets—one, two." The student counted one more time. "Five," she answered.

The next five sections of the book were about Benny's adventures as he spent the five pennies, one at a time. Each section used a repetitive rhyme to help students become familiar with the text. The first section was about a rose. Bozzato immediately sang, "How much is that rose in the window? The one with the long stem. How much is that rose in the window? I do hope that rose is for sale," connecting the story with a familiar song, "How Much Is That Doggie in the Window?" Students joined in as they recognized the tune. Next, she read the page in the story, which was also on a large chart for the students to read. They read it together twice, listening for rhymes.

" 'Will you sell me a rose?' asked Benny.
'Will you sell me a rose for a penny?'
'Yes, I will,' said Mrs. Hill."

The students, now familiar with the text, began to discuss the mathematics. If Benny buys a rose for a penny, how many pennies will he have left? "Show me with your fingers," said Bozzato. The students again quickly produced their number sentences on their hands, using their fingers. Orally, they told her, "Five take away one equals four." "Show me how many pennies Benny has now. Show it on the drum," suggested Bozzato, and one student drummed four beats. "Why?" she asked, and he replied, "He gave one to Mrs. Hill." Again, the teacher used talk to scaffold ideas and encouraged students to share the problem in various ways, with finger counting, drumming, song, rhyme, mathematical language, and discussion.

This format was repeated for the four other items—the cookie, the hat, the bone, and the fish. The students were totally engaged in remembering the words and tune of the song and the words, rhythm, and rhyme of the story and in solving the mathematical problem. In addition, all the students read each verse of the story with the teacher, creating with their fingers the number sentences $4 - 1 = 3$, $3 - 1 = 2$, $2 - 1 = 1$, $1 - 1 = 0$ and giving oral descriptions as the story progressed. Each time, the number of remaining pennies was drummed by a child.

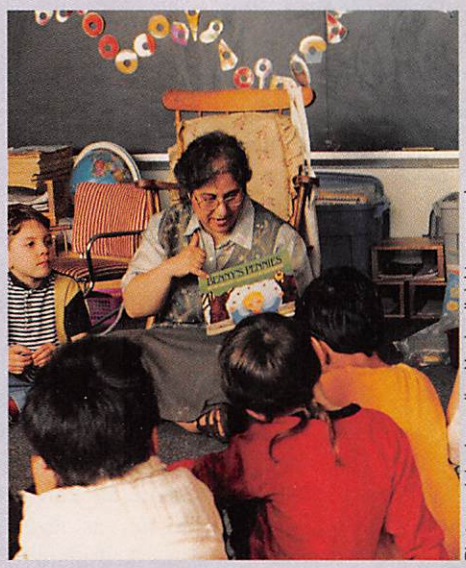
Reenactment and Reinventing through Drama: Continuing the Lesson

Throughout the week, Bozzato further developed this story in many ways. Students worked in small groups to read and physically dramatize the story by using the props. This dramatizing was a hands-on reenactment of the mathematics of subtraction. During these reenactments, the teacher also reviewed written number sentences to describe the subtraction, for example, $5 - 1 = 4$.

The story may also be changed—for example, by using different props—or extended—for example,

FIGURE 3

The class tried to predict the story from the picture on the cover.



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by spending pennies up to ten cents, using a nickel and making change, or selling items instead of buying them—depending on the students' understandings and interests. Thus, students can have many opportunities to reenact and reinvent the story of spending five pennies by reading, talking, dramatizing, singing, and writing in numbers and words.

The Home Connection

Research continues to connect student success to home involvement. Helping with homework, reading with children, and discussing school activities are three "types of parental engagement at home that are consistently associated with school performance" (Finn 1998, 20). With this research in mind, Bozzato organized a home book-bag program for her students. First, she carefully chose books for certain essential elements needed for ESL and emergent readers. In addition, before the books were taken home, Bozzato and her students interacted with the stories in numerous ways, by reading and recounting the story through rhymes, songs, and chants; with reenactments that used props and art; with reinventions through story changes and extensions; and with an analysis of grammar components.

Only after this intensive in-class preparation were the books sent home for independent, one-on-one sharing and writing with a parent or older sibling. The writing assignment was closely connected to the story but had, in addition, a home component that acknowledged students' personal interests and background. For example, a student might choose an item from home and tell a story about it by using a pattern from the book. In addition, the personal writing was shared, with the other students and the teacher back in the classroom. Sharing their writing again acknowledged the importance of the individual and the home for learning.

The home book bag for *Benny's Pennies* contained the book, a writing activity about spending a penny, and the props—a rose, a cookie, a hat, a bone, a fish, and five pennies. Students were encouraged to retell and read *Benny's Pennies* themselves, reenacting the subtraction story with the props. Parents were invited to help students as necessary, listen to the story, and become part of the dramatization. The students then used personal props, such as a teddy bear, a book, a package of gum, or different sums of money, to record their own stories and number sentences about pennies. Back in class, students shared their own versions of the story with the group.

Bozzato chose the writing activity for a book bag on the basis of the child's understanding and the time of the year. The activity could be closed or open-ended: a blank piece of paper, a classroom journal, or a specific worksheet. Whatever the choice, Bozzato found it important to give students and their families a chance to make personal meaning of the activity and to encourage independent thinking. For the *Benny's Pennies* book bag, Bozzato asked students to complete the activity shown in **figures 4–6.**

FIGURE 4

Latisha tried to use a different procedure—addition—and did not appear to understand the purpose of the activity.

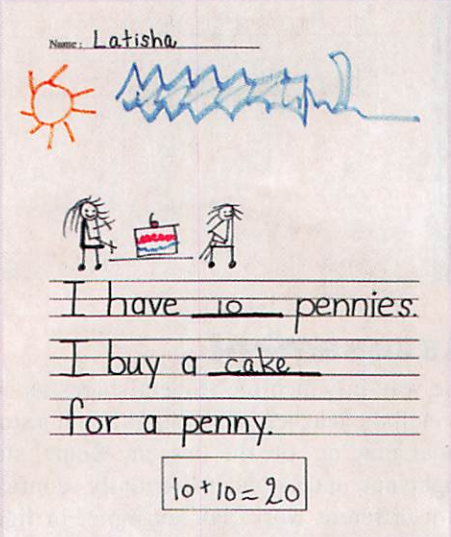


FIGURE 5

Sheldon demonstrated an understanding of the concept but needed to develop further some skills in printing numbers without reversals.

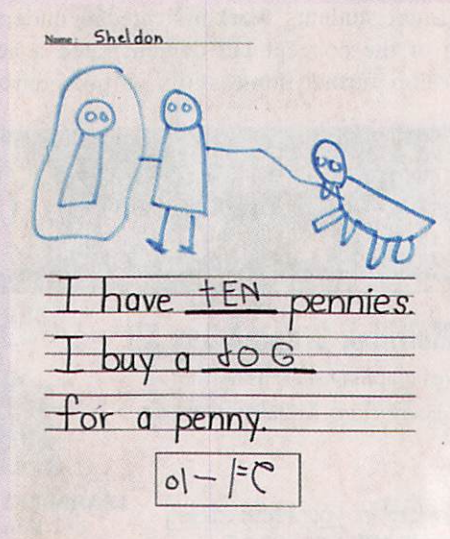
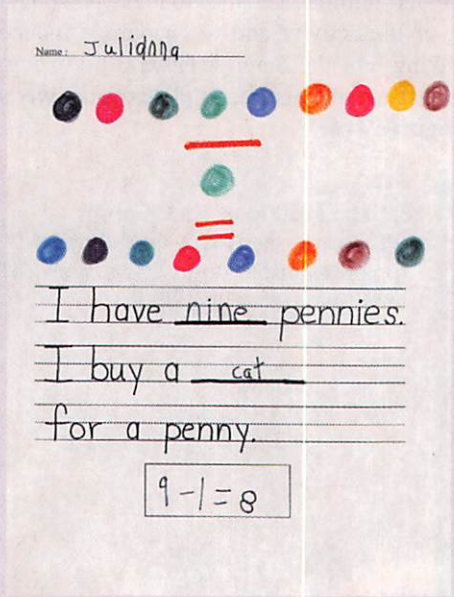


FIGURE 6

Julianna described a visual approach to writing a number sentence.



Assessment

The way in which the student shares an activity can tell the teacher many things about a student's understanding of the lesson. Some students might not understand the activity, confusing it with different work. For example, in **figure 4,** Latisha clearly had tried to use a different procedure—addition—and did not understand the purpose of the activity. She neither read the activity correctly nor wrote an appropriate number sentence to accompany her work, which told Bozzato that Latisha needed more time and opportunity to develop her understanding of the ideas presented in the lesson.

Some students' work indicated an understanding of the concept but demonstrated a need to develop further some skills or procedures; for example, in **figure 5**, Sheldon needed further experiences to help him print numbers without reversals. Although some students completed the activity as given, others became very creative, extending their thinking beyond the requirements of the activity. For example, in **figure 6,** Julianna described a visual-symbolic approach to writing a number sentence.

As the students read their work to the class the following day, the teacher asked more questions to assess further their literacy development and understanding of the mathematical concepts in the story. In fact, each sample told Bozzato much about risk taking and a particular student's understanding of the curriculum. Certainly, writing and reading their own sentences about money were ways for the students to expand their vocabulary meaningfully and gave them opportunities to apply their mathematical knowledge to other situations.

Developing Literacy in Mathematics and Language

Bozzato meticulously interwove literacy learning and mathematical understanding by using drama to create opportunities for her students to integrate language and mathematics, to make personal meaning, to relate the story to real-world activities, to make decisions, and to apply and extend their new knowledge. "The opportunity to participate in sociodrama and pretend play—situations in which children have to explicate verbally their plan for the scene, action, and objects—is an important prerequisite for successful performance in some routines which surround reading in the early grades" (Heath 1986, 168–69). It is also, I believe, an important prerequisite for mathematics literacy—for understanding and sharing mathematical concepts as students dramatize, talk, read, write, and draw to construct their understanding of mathematics.

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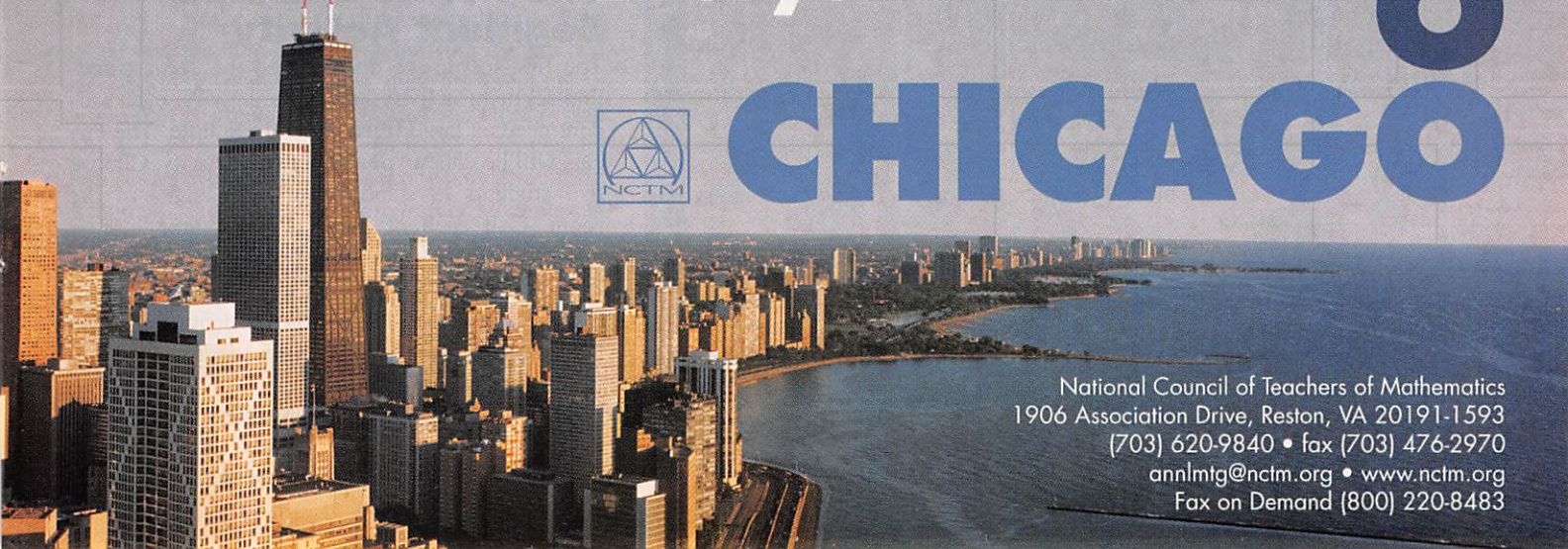

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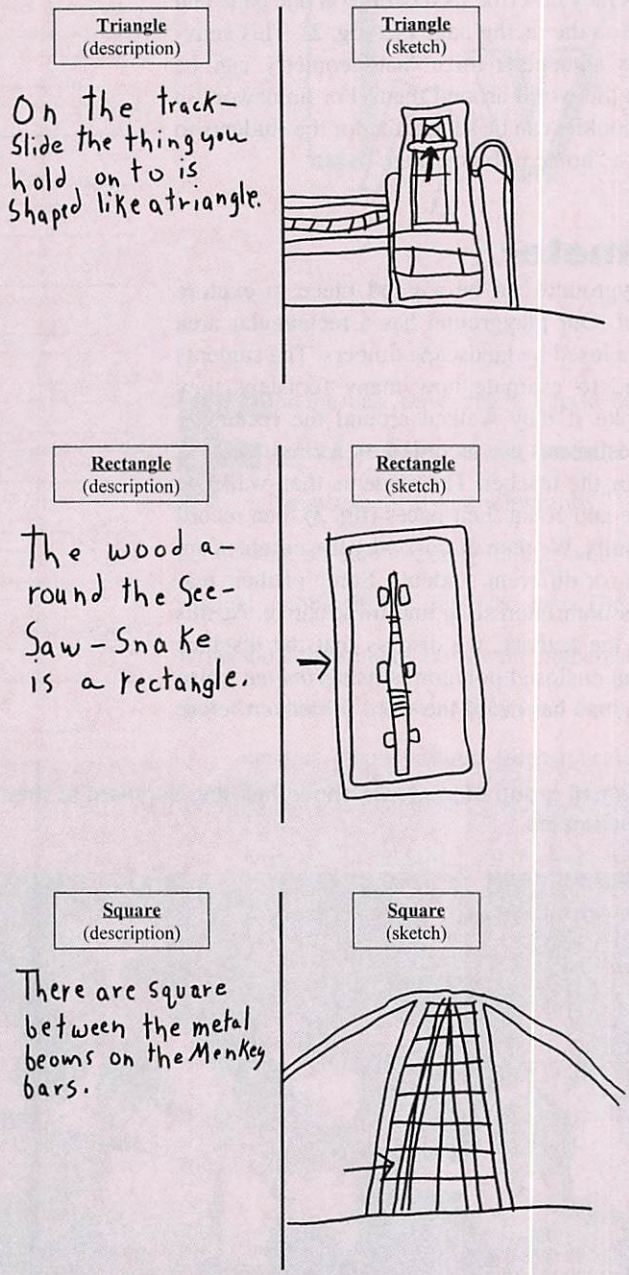
and how it was used. If you do not have a playground that allows this kind of access, either you or the students can create shapes on the floor with masking tape or on a tar surface with sidewalk chalk then measure them.

By using the rectangular area, we can discuss equivalent sides. For example, I might ask, "If it took me twenty-seven steps to walk down one long side of a rectangle, how many steps should it take me to walk down the other long side?" Some of the students will immediately reply, "Twenty-seven steps." I can then physically walk the distance for them and demonstrate that it is the same on both sides. Then we can turn our attention to the short side, and I ask, "If it takes me fifteen steps to walk down one short side, how many steps will it take me to walk the other short side?" Once again, after students share their answers, I can model the concept. When we go back into the classroom, I demonstrate the example on the overhead projector and then do some similar problems.

When working with perimeter, I also give students a perimeter measurement and ask them to use a ruler to draw a figure with that perimeter. We may start with a perimeter of twenty-four inches and move toward a measurement of thirteen inches. At the beginning, students think that this task will be easy, but they soon find that it is more complicated than they thought. By approaching perimeter in this manner, the teacher gets a different look at the thought processes of students. Given a shape with numbers along the edges, students may add these numbers without a clear understanding of why they did so. Therefore, students may appear to have a better understanding than they actually do. With this activity, if they do not understand perimeter, the deficiency quickly becomes apparent.

FIGURE 2

Students discuss and draw triangles, rectangles, and squares found on the playground.



Area and Volume

Getting students physically involved helps them avoid the confusion that sometimes occurs between the concepts of area and perimeter. After students have had experience with perimeter, create a rectangle on a portion of the floor using masking tape or borrow a rectangular mat from the physical education teacher. Ask the class to picture students lying inside the rectangle and estimate how many students it would take to cover the inside of the rectangle. Record various estimates, and then have groups of students lie inside the rectangle. Record the number of students needed

FIGURE 3

A student measures the perimeter of the rectangle by pacing it off.

Photograph by Stuart Robertson; all rights reserved

to cover the area. Discuss how this activity differs from the perimeter activity. During this discussion, the definition of area should emerge. Once again, key points should be recorded on chart paper and then incorporated into students' log entries.

If the topic has not come up by this point, discuss standard measures and their importance. This discussion provides a way of bridging the physical model that students made to one using small square tiles to cover the area inside a closed shape. Have students build their own models and record the areas to help them understand why area is recorded in "square" units.

Students can be physically involved in the early exploration of volume by simply standing in a large appliance box. Such boxes are often used as puppet theaters in primary classrooms. Once again, start the activity by having the class estimate how many students will fit inside the box then record their estimates. Have groups of students fill the box and record the number needed. Discuss the differences between this activity and the area activity. The students should realize that they are no longer just covering a flat area on the floor but are now beginning to fill space. The bridge from this physical model is using cubes to fill different-sized boxes, such as shoe boxes. This meaningful activity helps students understand why volume is recorded in "cubic" units.

Conclusion

This sampling of activities describes ways to get students actively involved with geometry and, when teamed with other methods of instruction, gives students multiple ways to experience these concepts. The activities do not require large amounts of preparation or special equipment, making them adaptable to many classroom settings and usable as a great starting point for those who wish to infuse their teaching with greater student involvement. The unit does not have to be done all at once. Start with an activity that is within your comfort level then continue to modify and expand your repertoire as your confidence level increases.

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Days of the Month: The Calendar of Mathematics

Hope Martin

These activities are designed to appeal directly to students. Students may work on the activities individually or in small groups. No solutions are suggested so that students will look to themselves as the mathematical authority, thereby developing the confidence to validate their work.

To help couple school mathematics and real-world mathematics, the activities this month are tied to historical events and holidays. Students are invited to engage in interesting activities that relate to May holidays and happenings.

These activities will also help students make connections among mathematics, other subject areas, and the real world. Historical data are used to help students see how mathematics relates to social studies and geography, and sports data are used to help students understand how statistics are used in our everyday lives. Students will enjoy relating mathematics to historical events and can find many other dates with which to extend their calendar of mathematics throughout the year.

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Hope Martin, hopem7930@aol.com, is a mathematics education writer and consultant in Skokie, Illinois. She taught elementary and middle school for thirty years.

Edited by **Karen Holly,** *kholly@kiwi.dep.anl.gov, 3728 West 67th Place, Chicago, IL 60629, and* **Sharon Rak,** *s.rak@cwix.com, 7907 West 81st Street, Bridgeview, IL 60455. Readers are encouraged to send manuscripts appropriate for "Math by the Month" to Deanna Rigdon and Jolyn Raleigh, Hunter Elementary School, 4351 South 4500 West, West Valley City, UT 84120.*

WEEKLY ACTIVITIES

DAYS OF THE MONTH — MAY
K–2 — 1999

3 — **May 5th is the day that the people of Mexico** celebrate their victory over the French army in Mexico. It is called Cinco de Mayo, the Fifth of May. Explore items that relate to "five." Have each student draw or write about something based on the number 5. Have students count the sum of all the sets of five on their pictures. What do students discover about five and its multiples?

10 — **On 14 May 1686,** Gabriel Fahrenheit was born. He invented the mercury thermometer in 1714. Keep track of the high and low temperatures for this week. What was the difference in temperatures each day? Which day had the greatest difference? What was the highest temperature reached? The lowest? Design a graph to record the temperatures. Write a few sentences to describe the temperatures and the meaning of your graph.

17 — **On 16 May 1866,** the United States minted and circulated the first five-cent piece. In honor of the first nickel, read *A Quarter from the Tooth Fairy* by Caren Holtzman (1995) and determine in how many different ways you can make change for a quarter using pennies, nickels, and dimes.

24/31 — **On 30 May 1783,** the first United States daily newspaper, *The Pennsylvania Evening Post,* was printed in Philadelphia, Pennsylvania. Create your own class *Math Newspaper.* Have each student create one puzzle, write an original story problem, or design an interesting number pattern for the newspaper. Decide how often to publish the newspaper.

WEEKLY ACTIVITIES

DAYS OF THE MONTH: 3–4 **MAY 1999**

3

On 1 May 1931, the Empire State Building was dedicated in New York City and was, for its time, the tallest building ever constructed. It has 1860 steps. Work with a partner to measure the height of five different steps around your classroom or school and find their average height. Using this information, can you calculate the approximate height of the Empire State Building, not including its tower, which has no steps?

10

The second Tuesday in May is National Bike to Work Day. Work with a group of friends to measure the diameter of the front wheel of a bicycle. Roll the wheel one complete turn, and measure the distance traveled. Do the same with other bicycles. Create a table, and record the measurements. Can you find a pattern in the relationship between the diameter and the distance the wheel traveled in one turn? Does your pattern work for other circles?

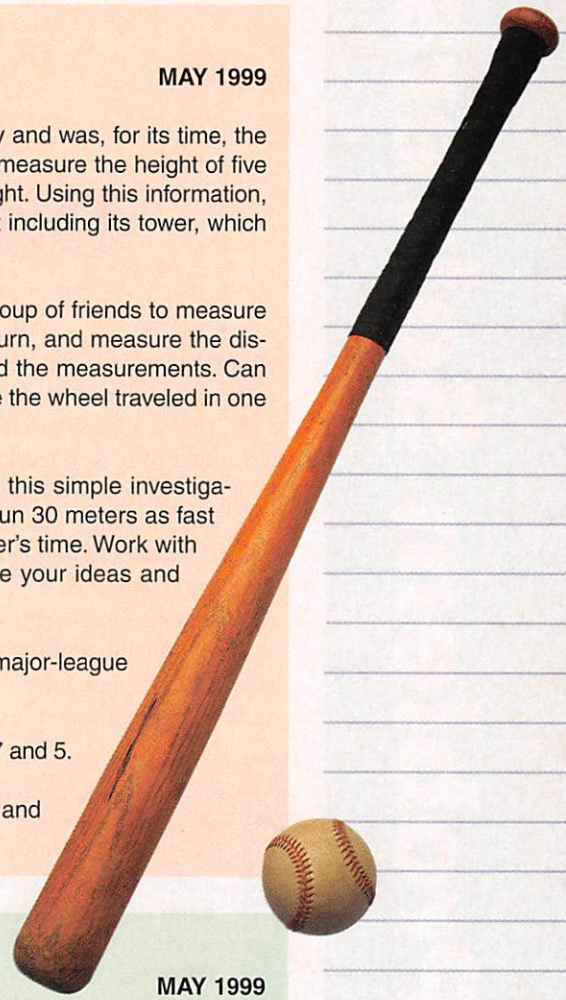
17

The first running of the Kentucky Derby occurred on 17 May 1875. In this simple investigation, you will explore your running speed. Have each person in the class run 30 meters as fast as he or she can, and time each one with a stopwatch. Record each runner's time. Work with a partner to examine the data and discuss how you could graph it. Share your ideas and graphs.

24/31

Babe Ruth hit the 714th home run of his career on 25 May. It was his last major-league home run as well as his last hit. To learn the year, solve this puzzle:

- The two-digit number formed by the tens-and-units digit is a multiple of both 7 and 5.
- The hundreds digit is three times the tens digit.
- The sum of all of the digits is the same as the number of months in one and one-half years.



WEEKLY ACTIVITIES

DAYS OF THE MONTH: 5–6 **MAY 1999**

3

Known as Cinco de Mayo, May 5th is celebrated by the Mexican people to recognize the anniversary of the Battle of Puebla in 1862, when Mexican troops defeated the French army in Mexico. To help celebrate Cinco de Mayo, try this puzzle: Using five 5s, add, subtract, multiply, or divide to obtain answers of 1 through 10. For example, $(5 + 5 + 5 + 5) \div 5 = 4$. Be sure to use parentheses, if necessary, and follow the order of operations.

10

Gabriel Fahrenheit, a German physicist, was born on 14 May 1686. He invented the mercury thermometer. The United States uses the Fahrenheit (F) scale to record temperature, but most of the world uses the Celsius (C) scale. The highest temperature ever recorded on earth was 57 degrees C in Death Valley, California. To convert this temperature to degrees Fahrenheit, use this formula: $F = (9/5 \times C) + 32$. Research other record temperatures, and convert them from Celsius to Fahrenheit or vice versa.

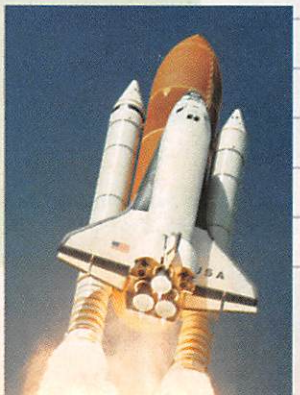
17

Amelia Earhart was the first woman to fly solo across the Atlantic ocean. She left Newfoundland, Canada, on 17 May 1932 and flew 2026 miles in 20 hours and 40 minutes to land in Wales on 18 May. In honor of her flight, work with your classmates to build paper airplanes. Decide on a useful unit of measure, and record the distance that each airplane flies. Find the average distance of three trial flights to determine which airplane flies the farthest.

24/31

Sally Ride, the first United States woman in space, was born on 26 May. To discover the year that she was born, solve this puzzle:

- If the tens digit were four greater, the year would be a palindrome.
- The two-digit number formed by the tens and units digits is a multiple of 3.
- The sum of the thousands and hundreds digits is equal to twice the tens digit.
- The sum of all of the digits is 16.



Problem Solvers

Darts, Anyone?

Melfried Olson

The goal of the "Problem Solvers" department is to foster improved communication among teachers by posing one problem each month for K–6 teachers to try with their students. Every teacher can become an author: pose the problem, reflect on your students' work, analyze the classroom dialogue, and submit the resulting insights to this department. Remember that even student misconceptions are interesting.

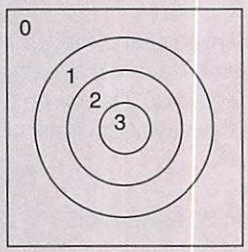
The following problem is suggested for students in grades 1–6, although it may be appropriate for students in other grades as well.

PROBLEM

Ted and Kate were making a dart game for their children. They wanted to have three rings marked with number values. Ted made a board like the one shown. When three darts are thrown, what is the lowest score possible? What is the highest score possible? How many different point totals are possible using three darts?

Kate did not like this dart board because it permitted only ten point totals. That is, using three darts, a person could score 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 points. Show how each total can be obtained with three darts.

Kate wondered whether it was possible to label the rings so that a player could reach a higher score with three darts. She wondered whether all the numbers from 1 to 10—or even higher—were possible. What suggestions would you give for labeling the rings?



Please spend some time discussing this problem with your students. Although the students may need help understanding the problem and the constraints under which they are to work, please avoid giving too much guidance. Discuss with your students the need for having a "zero region" on the dart board. Collect students' work, and jot down notes about discussions that occurred and the variety of students' solution processes.

Where's the Math?

This problem has many mathematical facets. It involves mental computation, flexibility with numbers, and logical reasoning. Children may also have to struggle with the notion that if rings are labeled 1, 4, and 7, more values may be obtained but that the longest string of consecutive numbers starting at 1 is from 1 to 9, just as with Ted's original board.

Edited by **Melfried Olson, Lynae Sakshaug,** *and* **Judith Olson,** *Mathematics Department, Western Illinois University, Macomb, IL 61455*

View this task as more than an exercise for which students are seeking a correct answer.

As you reflect on your experience with the problem, keep in mind these questions:

- Did the students understand the idea of totaling points and trying to get the longest consecutive string?
- What strategies did the students use? Did they use mental computation when working on the problem? Did they discuss how a value could be obtained in different ways?
- What did you discover about children's number sense as you watched them work on this problem?
- How well did the students explain their reasoning? How well did the students persist in looking for more solutions?
- Did the students relate this problem to any other problems that they have investigated? Did you or the students pose any extensions for this problem?
- How could you assess students' work on this exploration?

Please send us your thoughts and reflections. Include how you posed the problem and samples of students' work. Please send your results with your name, grade level, and school by **1 July 1999** to Melfried Olson, Mathematics Department, Western Illinois University, Macomb, IL 61455.

Your analyses will help other teachers better understand children's capabilities and their thinking about mathematics. Selected submissions will be published in a subsequent issue of *Teaching Children Mathematics* and acknowledged by name, grade level, and school unless otherwise indicated. If you would like an acknowledgment and a set of solution strategies, send a self-addressed, stamped envelope to Melfried Olson at the address shown.

(Solutions from previous issues begin on page 534.)

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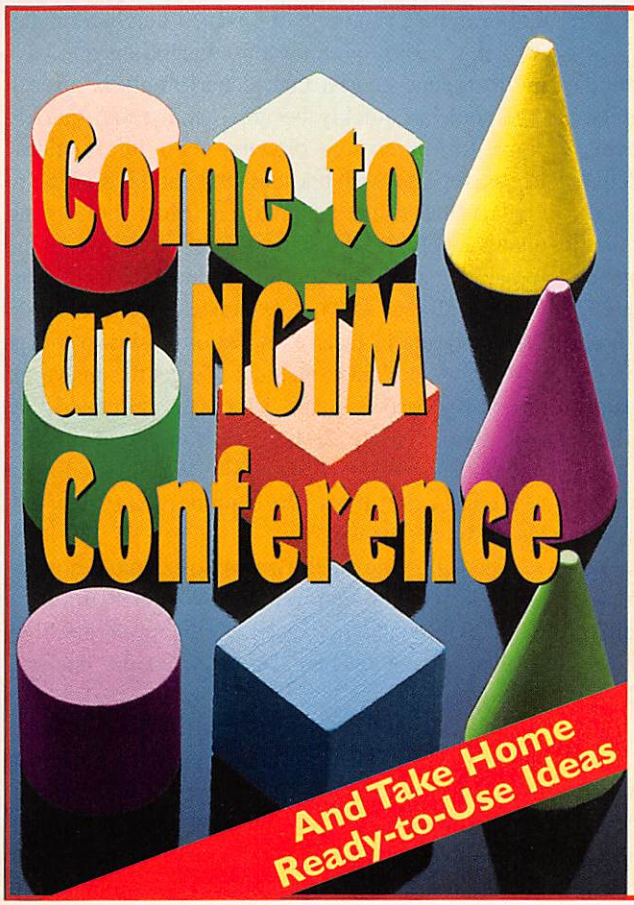

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(Continued from page 533)

Responses to the "How Many Times Can You Take One-Half?" Problem

The problem appearing in the September 1998 "Problem Solvers" section was stated as follows:

> To encourage people to attend a special concert, radio station MATH decides to give out 1242 free tickets. The station decides to distribute tickets in the following manner. Each day it will give away half of the tickets in its possession. How many days will it take before MATH has only one ticket left to give?

The responses indicate that this was a good problem for fifth- and seventh-grade students. Interestingly, the responses from both grade levels included four common threads. First, the students needed time to think about what the problem was asking and needed to work to understand the first step in the problem-solving process—understand the problem. Second, the students needed to determine how to deal with a remainder when dividing by 2 and to develop a strategy, the second step in the problem-solving process, for handling remainders. Some students thought that when they obtained a remainder, they could no longer give half and, hence, the problem was solved. Others realized that a decision had to be made—either give away the remainder, keep the remainder, or alternate—depending on the situation and the numbers involved. Third, once a strategy was chosen, a solution based on that strategy was found and the process was recorded. In so doing, students employed the third step in the problem-solving process. Fourth, after choosing a strategy and obtaining a solution, students wrote about what they did. This step appeared to challenge students at each grade level, since they found it difficult to use the fourth step in the problem-solving process—look back and reflect. These observations remind us that students need assistance and experience with the problem-solving process. We also found that one problem can be used successfully at different grade levels.

Dan Brown, a fifth-grade teacher in Florida, New York, noted two common misconceptions. One was that "students thought the problem was answered when they came upon the first remainder" or "that the answer would be two days because 1242 can be given out half on day 1 and half on day 2." After dealing with the misconceptions and having a student suggest that a chart might be useful, "the students quickly became excited and quickly constructed charts to organize their information." Brown reported that his students found it difficult to explain their solutions in writing. Most students did obtain 10 or 11 days, depending on their use of the remainder and their recording process. Some students used 1242 as day 1, which resulted in 11 days. One particularly clear explanation is shown in **figure 1.** This student explained, "To get the answer of ten days I made a chart. I took the number 1,242 and divided it by 2. I didn't want to deal with remainders, so I gave them away. When I finally got to the last ticket it was the tenth day, so I knew that had to be my answer."

The following week Brown posed another problem that would again require the students to use a table. After students clarified the problem, they applied what they had learned from the one-half problem.

The fifth-grade students in Rhonda Kinnish's class in Midland, Michigan, responded similarly to Brown's students. They employed similar strategies, drew fewer tables, but provided more elaborate written explanations. They also struggled with the remainder. One student, in an effort to make the

problem manageable, reported, "First I took a calculator and divided 1,242 by 2. Then I divided the answer [621] by 2. Here I ran into a problem. My answer was 310.5. I decided to round it to 310. I did this because they can't change the amount of tickets and that is the only other option. I rounded it to 310 because it can be divided by 2. I then divided this by 2, getting 155. I then divided that by 2 and ran into the same problem, 77.5. This time I rounded up, because 78 can be divided by 2. Again I divided by 2, getting 39. When I divided this by 2 I got 19.5. I rounded again to 20 for, again, it can be divided by 2. Now I only have 10 tickets left. Dividing by 2, I got 5. Divided that by 2 and [rounded to] get 2. Divide[d] this by 2 [to] get 1. We have done this ten times therefore they had 1 left on the 10th day." Clearly, this student understands the problem and realized the complexity of dealing with the remainder. Also, the student was looking ahead when deciding how to round. Knowing that halving an even number would be best, rounding up or down was determined by which would give an even number. As the issue of whether to round up or down in this situation is arbitrary, this solution is quite reasonable.

When Ione Hesch, a seventh-grade teacher at Rockridge Junior High School (Taylor Ridge, Illinois) posed this problem, she asked the students to first predict the number of days that it would take. The estimates ranged from 8 to 59 days. That only 10 days are needed may surprise people, and it reminds us that if we had started with 2484 tickets, or twice the original number of tickets, only one more day would have been needed.

As might be expected, those who understood the problem and provided correct solutions used strategies similar to those of the fifth-grade students, but their work was more elaborate and their explanations more detailed. Many made the same errors of either listing 1242 tickets as the first day or specifying the day with two tickets left as being the last day. These minor problems with recording did not detract from how well the students demonstrated their understanding. Some created clever schemes to deal with the remainder. One student saved all the remainders "until the end" and then added the remainders all at once. This plan resulted in having tickets to give away until day 13, when only one ticket was left. Another student used a calculator and recorded the decimal value obtained by successive divisions by 2. A visual approach was used by Ethan **(fig. 2).** Ethan obtained the solution of ten days, but he made several errors in getting that answer. However, the graphical representation that he used gives a visual perspective to the process and, if done correctly, demonstrates how quickly one gets to one ticket by repeatedly halving the amount. The graphical representation can also show why starting with 2484 tickets adds only one day to the process.

Some of Hesch's comments regarding the difficulty that her students first encountered were similar to those shared by the fifth-grade teachers: "A few students did not know how to find half of a number

FIGURE 1

Sara

1242

Gave Away	Day	Tickets Left
621	1	1242 - 621 = 621
311	2	621 - 311 = 310
155	3	310 - 155 = 155
78	4	155 - 78 = 77
39	5	77 - 39 = 38
19	6	38 - 19 = 19
10	7	19 - 10 = 9
5	8	9 - 5 = 4
2	9	4 - 2 = 2
1	10	2 - 1 = 1

FIGURE 2

Name Ethan

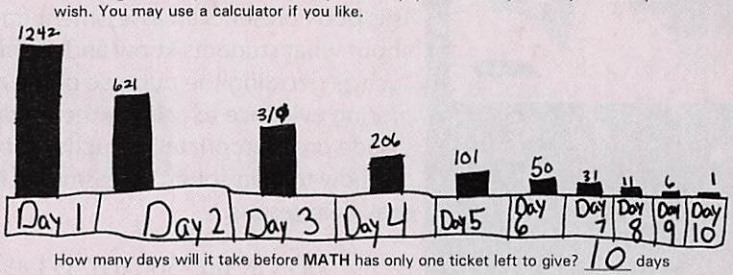
Problem:

To encourage people to attend a special concert, radio station **MATH** decides to give out 1242 free tickets. The station decides to distribute tickets in the following manner. Each day it will give away half of the tickets in its possession. How many days will it take before **MATH** has only one ticket left to give?

Before you start, predict the number of days you think it will take before only one ticket is left?

Your prediction: 10 days

In the space below, show how you keep track of the tickets given away/tickets remaining. You may use the back or your own paper. You may use any method you wish. You may use a calculator if you like.

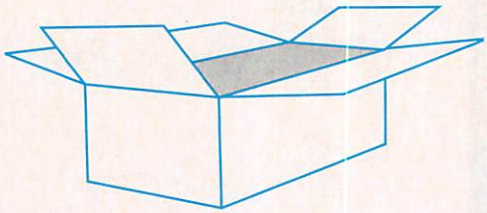


How many days will it take before **MATH** has only one ticket left to give? 10 days

Tell how you see the original number of tickets (1242) relates to the number of days necessary to end up with one ticket left?

I formed my own sort of pattern. When you take half of 1242 its 621. Some kids think its hard to take half from a odd number so on Day 3 I saved a ticket for day 10.

MOVING?



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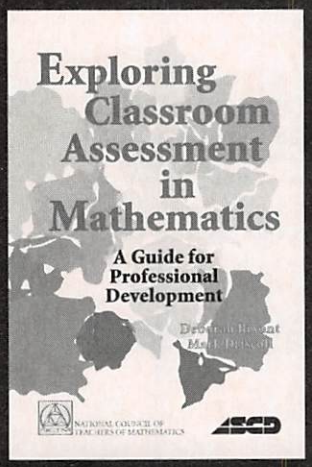
and had to be told that dividing by 2 was one way of finding half of a number. Some students did not at first understand that there would be a smaller number of tickets to give away each day. . . . Putting mathematical thoughts into writing was new and difficult for many." Although these comments perhaps reflect "immature" reasoning at the seventh-grade level, this type of investigation allows the teacher to note the level of reasoning that students possess.

From these explanations and comments, we observe that students—

- can use the same task successfully at several grade levels.
- need to experience problems in which they have to make decisions.
- need work with strategies.
- need experiences with number relationships, such as that dividing by 2 is the same as finding one-half.
- need prodding to get beyond the initial feature—"If I get a remainder, I am done"—but can make intelligent decisions—"Round up sometimes, and round down other times."

Teachers can learn from the work of their students. Although it may be hard for students to put their thoughts on paper, teachers should make the effort to see that students have these experiences. ▲



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By Deborah Bryant and Mark Driscoll

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Shaping the Standards:
Continuing the Journey

Over the past few months, some lively dialogue has occurred about the discussion draft of NCTM's updated standards, titled *Principles and Standards for School Mathematics,* as a part of providing the writers feedback in preparing the final version. The "Shaping the Standards" articles in this journal have been an interesting part of that discussion.

It has been exciting to observe the creative ways that our members have used the draft to forge interesting conversations, not just about the messages and format that our updated Standards should take but also about important issues in mathematics teaching. We hope that these discussions will continue to occur in the coming months and years. Please take a moment to share some of your experiences:

- What formats for discussion did you find to be particularly worthwhile? How were the suggestions from the articles in this series useful in discussions with your colleagues?
- What were the particular advantages in discussing a draft instead of a final document? How could these advantages be preserved when the final document is released?

In addition to your comments about these questions, we encourage you to send in any comments about the draft before 15 May 1999—including those comments from last month's article that you have been meaning to send in, those notes from your school's discussion group that are still sitting on your desk, and any final thoughts that you would like to share with the writers as they prepare the document during summer 1999.

This article is the final installment in the series for this school year. Beginning in fall 1999, this journal will include several articles summarizing the feedback that we have received, looking forward to the release of the final *Principles and Standards for School Mathematics* in April 2000. Thank you for your feedback through this and other venues.

As usual, you can submit your comments through NCTM's Web site (**www.nctm.org/standards2000**), e-mail (**shapingstandards@nctm.org**), or mail (Standards 2000, NCTM, 1906 Association Dr., Reston, VA 20191). Electronic responses are encouraged when that option is available. ▲

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Looking for a hot topic of discussion? Try temperature. Although many applied-measurement activities are used in classrooms today, most focus on distance or position. The fundamental concepts of temperature and the change in temperature over time are often overlooked.

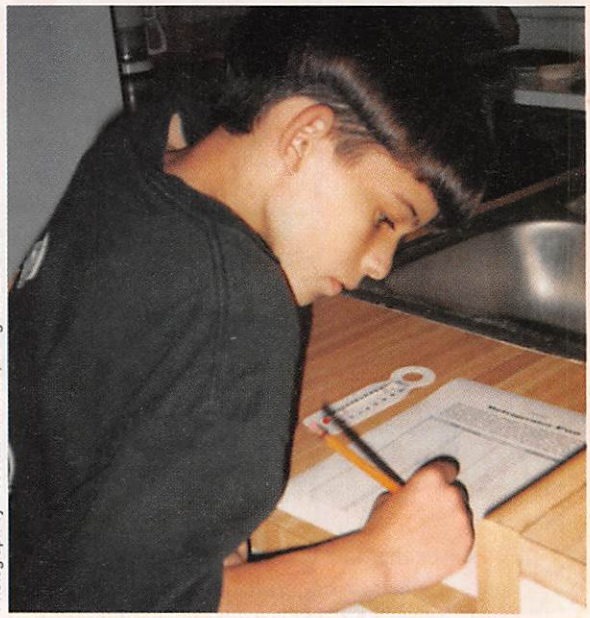
Students need experience with measurement. The NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) addresses this need at both the K–4 and 5–8 grade levels and calls for students to attain an understanding of various types of measurement. With concrete experiences, students can easily gain this understanding and then use measurement to represent the phenomena of everyday experiences. When students have collected measurements, teachers can easily guide them to use tables and graphs to neatly organize and visualize their data. This process helps students determine patterns and identify trends.

Deborah A. Moore

The qualitative idea of temperature is something that most young children can understand easily. They can tell whether something is cold or hot. They know that the temperature in summer is hotter than it is in winter. A wealth of childhood temperature experiences range from "Don't touch the stove; it is hot!" to the temperature of the porridge in the story of *Goldilocks and the Three Bears*.

Temperature activities present a meaningful background from which teachers may introduce mathematical concepts. This article offers for many grade levels measurement and graphing activities that use temperature as a common theme.

Deborah Moore, d_moore@rumac.upr.clu.edu, teaches at the University of Puerto Rico at Mayagüez, Mayagüez, PR 00681-9018. She is interested in integrating mathematics and science to provide a basis for the early introduction of important mathematical concepts.



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Temperature throughout the Mathematics Curriculum

Temperature activities are relevant all through the school mathematics curriculum. Teachers can have primary students touch items and measure their temperatures. For example, students can compare a bowl of soup and a glass of milk. If both have just been poured, they know that the soup is hotter than the milk. The teacher can allow students to feel each container and then demonstrate for them how to measure each item's temperature. Once students grasp the attribute of temperature, they can sort from coldest to hottest various pictorial situations that involve weather, such as children building a snowman or swimming.

Some Li

In grades 2 and 3, when students have developed a sense of how warm a certain temperature is, they can match specific temperatures to pictorial situations. For example, teachers can show students pictures of an ice cube, a cookie, a glass of ice water, and a cup of hot chocolate with steam rising and ask them to match these items to given temperatures. To encourage the use of the metric system, teachers can ask at which temperature they prefer to drink lemonade: minus 5, 5, or 25 degrees Celsius? Students then form and justify guesses.

By grades 4 and 5, students should be able to gather daily outdoor temperatures and make monthly charts. They can use these charts to predict weather trends. As an extended activity for students who have learned the concept of average, the teacher can post a bar graph showing the daily high temperatures for a week. When all the data from a week are posted, students can then determine the average high temperature for that week. The average temperature can then be calculated by the students and drawn on the graph as a horizontal line (see **fig. 1**). This visual representation allows students to see that the average must be between the two extremes. It also affords an excellent opportunity for the teacher to introduce other statistical concepts, such as range. Once a weekly average is recorded, students can hypothesize what might happen the next week and why.

Promoting Graphical Thinking with Temperature

Once students have had sufficient experiences with temperature, they can learn about the role of temperature and its relationships with time in our physical environment. Activities that investigate the change in temperature over time build the needed foundation for one of the most important concepts in calculus: rate of change.

A nice activity for fourth graders is suggested in the *Standards* document (1989) to help foster data-analysis skills. It involves comparing the local daily temperature with the daily temperature in Sydney, Australia. By having students create line graphs for each month and then calculate the average temperature for that month, they are led to observe that the temperature in Sydney grows warmer as it becomes colder in the United States. Students not only are engaged in data analysis but also are learning the effect of the earth's tilt on the seasons. This observation is a timely topic for discussion as the Olympic games in Australia approach.

The activities described lay a solid foundation for the notion of temperature as a function of time. Building on these experiences, students in grades 5 and 6 will be ready to investigate the change in temperature over time in the context of heating or cooling. An excellent homework activity for students at this level is to monitor temperature change as measured by a thermometer that has just been removed from a refrigerator.

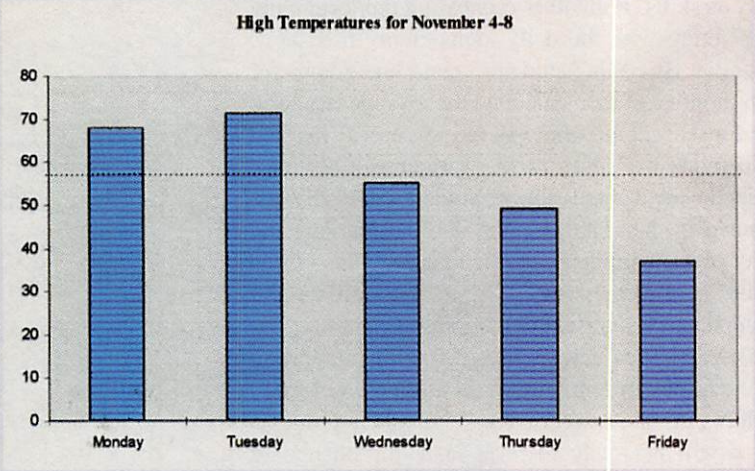
Before they begin the activity, have students predict the behavior. A discussion of what should happen is easily prompted by questions like the following: Will the temperature increase or decrease when the thermometer is removed from the refrigerator? Why? Will it ever stop changing? Why? Where will it stop? Why? This activity affords an excellent opportunity for students to make conjectures and discuss with one another what outcomes are likely to happen.

In this assignment, students form

Promoting Measurement and Graphical Thinking by Using Temperature

ke It Hot!

FIGURE 1



The average of the daily high temperatures of 68 degrees, 71 degrees, 55 degrees, 49 degrees, and 37 degrees is 56 degrees, as shown by the horizontal line.

a data table and then create a graph representing the behavior of the warming thermometer. If thermometers are available to take home, students can complete this activity at home with their parents. It does not make a mess and does not take very long. Since it involves familiar objects, parents should be willing to participate. The only items needed are a thermometer, a refrigerator, and a clock or watch marked with five-second intervals. This activity is best done at home with at least two people—one timekeeper and one thermometer reader. It is often easier if a third person records the data.

The teacher instructs students to leave the thermometer for one hour in the coldest part of the refrigerator that is easily accessed. When they are ready to begin the experiment, they should have their clock, pencil, and paper readily available. Depending on the previous experiences that the students have had making tables and charts, the teacher may want students to use worksheets, like **figures 2** and **3,** with a table and a set of axes to

FIGURE 2

Worksheet for student data collection

Refrigerator Fun

Place a thermometer in your refrigerator. Put it in a place that is very cold but easy to reach. Leave it there for 1 hour. While you are waiting, find a clock or watch that has a mark every 5 seconds, a pencil, and a parent or older relative to help you. You are going to read the thermometer and write down temperatures. Your helper is going to keep time. When you are ready, quickly take the thermometer out of the refrigerator and call out the temperature. Then every 5 seconds, the timekeeper will say, "Now!" and you will call out the temperature and write it down. Do this until the table is completed.

TIME (in seconds)	TEMPERATURE (°F)
0	
5	
10	
15	
20	
25	
30	
35	
40	
45	
50	
55	
60	
65	
70	
75	
80	
85	
90	

complete rather than have students create them on their own. The teacher gives students the following instructions:

> Decide who is going to write down the temperatures; it could be either the timekeeper or a third person. Open the refrigerator door, immediately pull out the thermometer, and state the temperature. Place the thermometer on a table or countertop as soon as you remove it from the refrigerator. Every five seconds, the timekeeper should say, "Now!" and the thermometer reader should say the temperature shown on the thermometer out loud so that it can be recorded. Do this for one and a half minutes.

The behavior in the experiment is even more pronounced when the thermometer is placed in the freezer; however, not all classroom thermometers are able to measure temperatures that low. Check on the range of the thermometers that your students will be using before assigning a freezer activity.

Once the activity is completed, have students bring their tables and graphs to class and describe them to the other students. Encourage them to distinguish between the sections where the graph increases rapidly and those where it increases slowly. Ask why some graphs started and stopped at different numbers than others. Ask what would happen if they performed the same experiment outside on a very hot day or a very cold day. How would the results differ? See **figure 4** for completed worksheets.

This activity paves the way for more difficult concepts. By engaging in this activity before being introduced to words like *concavity,* students are able to relate to the physical phenomenon without being encumbered with definitions. Even at this

FIGURE 3

Worksheet to graph the collected temperature data

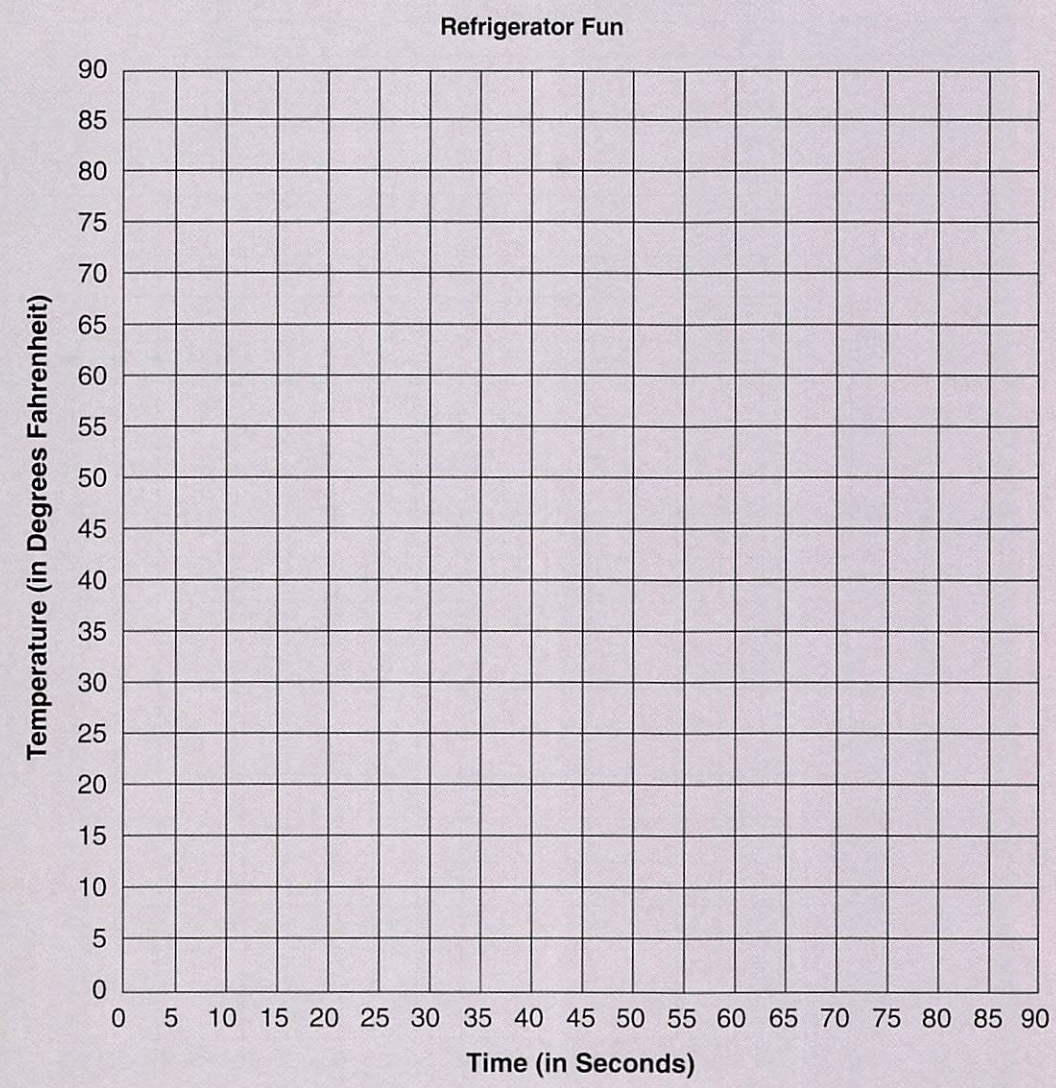
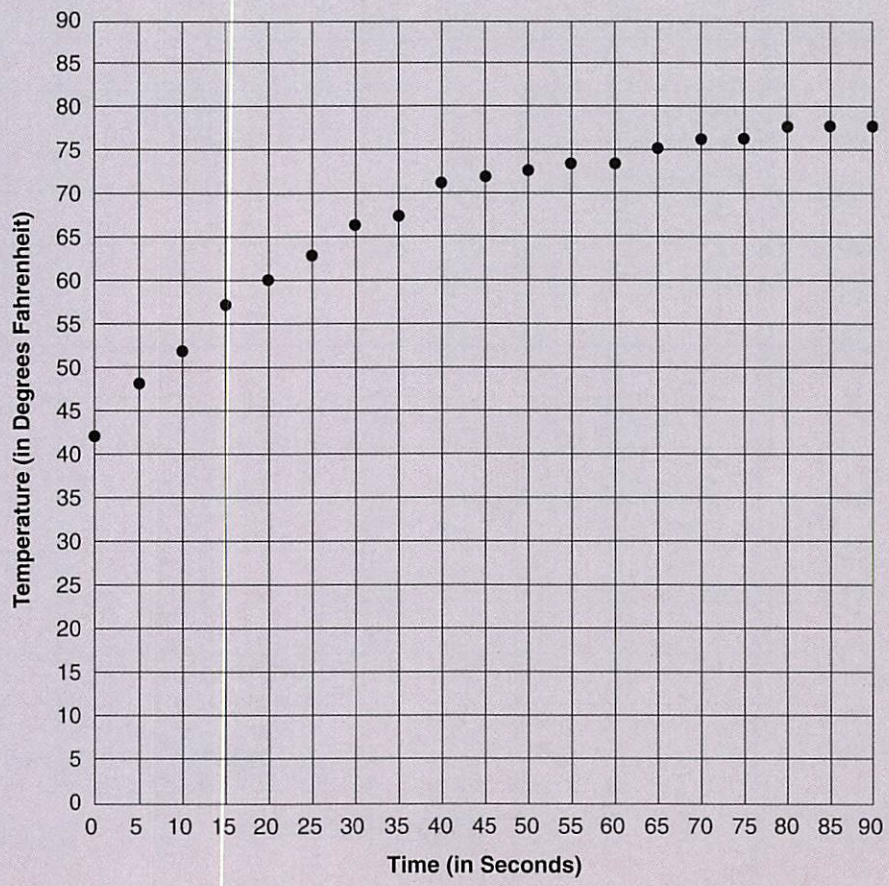


FIGURE 4

A completed worksheet and graph

Refrigerator Fun

TIME (in seconds)	TEMPERATURE (°F)
0	42
5	48
10	52
15	57
20	60
25	63
30	66
35	67
40	71
45	72
50	73
55	74
60	74
65	75
70	76
75	76
80	77
85	77
90	77



young age, students can see that the temperature increases—"goes up" in their language—rapidly at first then less rapidly until it reaches room temperature and no change is apparent. The analysis of the graph and the rate at which it increases prepares for learning at the next level, since the concept of change—more specifically, the rate at which a function changes—is fundamental to the conceptual underpinnings of calculus.

One exceptional demonstration (Brueningsen et al. 1994) to introduce the idea of temperature change before students engage in the homework activity involves the Calculator-Based Laboratory (CBL) system by Texas Instruments and either a projectable graphing calculator or a computer. The CBL unit is a portable device that directly downloads to a graphing calculator or computer the data obtained from experiments employing its various sensors that measure temperature, distance, light intensity, and so on. However, if no such technology is available, this demonstration can be done with a candy thermometer. Tell students that you are going to place the temperature sensor of the CBL into a pot of nearly boiling water and then remove it. Encourage students to hypothesize how the temperature will change over time. After the students have predicted what will happen, perform the experiment. Since the calculator or computer and the CBL are linked, a graph will show the temperature versus time while the CBL collects data.

Temperature as a Link between Mathematics and Science

Many more activities use temperature to link mathematics and science. Two such activities (Stenmark, Thompson, and Cossey 1986) involve the scientific concepts of friction and light absorption. The first activity begins with students' feeling a piece of wood and a piece of sandpaper. Then, after rubbing the wood with the sandpaper for a while, students touch each surface again to feel the temperature increase as a result of the friction. Since no numbers are involved in this activity, it presents an opportunity for students in earlier grades to compare temperature by using only their senses.

The second activity is more suitable for slightly older elementary students. It requires two thermometers, a piece of black construction paper, a piece of white construction paper, and a pleasant sunny day. Place the black paper and the white paper side by side in full sun. Put a thermometer under each. Have students predict what will happen and then check the temperature of each thermometer after five minutes and again after thirty minutes. Besides being a practical application of measurement, the comparison of two numbers, and subtraction to determine the amount of change in, and the difference between, the temperatures, the results of this activity lead to the concept that certain colors absorb more light than others.

Conclusion

Not only is temperature basic to humans' intrinsic tactile sense, but it also often governs our way of life. Temperature affects such diverse things as storing food safely, taking comfortable baths, determining which plants will grow, and deciding which clothes we should wear. The temperature activities in this article allow students to explore and hypothesize. They allow students to measure, record, and analyze real-world phenomena.

Many other activities also take advantage of students' prior experiences with temperature. For instance, temperature activities can serve as a concrete introductory model for negative numbers, and temperature can be a practical model for unit conversions from Fahrenheit to Celsius.

The concept of temperature is fundamental and deserves more emphasis throughout the school curriculum. Besides serving as a natural link between mathematics and science, it also offers a logical way to encourage students to think graphically. As the saying goes, the heat is on.

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Teaching and Learning Creatively: Using Children's Narratives

Ana Maria Lo Cicero, Yolanda De La Cruz, and Karen C. Fuson

The Children's Math Worlds project seeks to integrate students' social, emotional, and cultural experiences into classroom mathematics. For seven years, we have been developing in classrooms a conceptually challenging research-based mathematics curriculum called Children's Math Worlds (CMW) for kindergarten through grade 3. We build on the individual experiences, interests, and practical mathematics knowledge that diverse children bring to our classrooms. Our collaborative research project has been, and is being, carried out in urban schools of underrepresented minorities, mostly Latino English-speaking and Latino Spanish-speaking children, and in English-speaking upper-middle-class schools to ensure that our work crosses socio-economic boundaries. The CMW family component was described in De La Cruz (1999). See that article for data concerning the excellent comparative performance of CMW children.

In this article, we focus on two central, related activities of CMW: (1) linking mathematical activities in the classroom to children's mathematical experiences outside of school and (2) creating a rich and sustained environment for learning to write, solve, and explain ways of solving word problems. Solving word problems has traditionally been difficult for many children, especially those for whom English is a second language. Word problems are often neglected and not assigned to these children. We have found that centering our classes on such problems, using problems of increasing difficulty, and supporting language use by children enable them to solve word problems readily. For more details, see Fuson et al. (1997). In our following description, we weave in voices of some of our classroom teachers to comment on various aspects of teaching using children's lives.

Ana Maria Lo Cicero and Karen Fuson, fuson@nwu.edu, teach at Northwestern University, Evanston, IL 60208. Yolanda De La Cruz, ydelacruz@asu.edu, teaches at Arizona State University West, Phoenix, AZ 85069. Lo Cicero works with children and teachers to develop classroom activities that support their mathematical and personal development. Fuson studies how children think mathematically and designs learning activities to build on children's thinking so that all children can reach their potential. De La Cruz studies how to overcome failure in mathematics learning among Latino students.

Edited by **Tad Watanabe,** *Towson State University, Mathematics Department, Towson, MD 21204. Readers are encouraged to send manuscripts appropriate for this department to the editor.*

The research reported in this paper was supported by the National Science Foundation (NSF) under grant no. RED 935373, the Spencer Foundation, and the James S. McDonnell Foundation. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of NSF, the Spencer Foundation, or the James S. McDonnell Foundation. For more information about the Children's Math Worlds project or the curriculum, contact Karen C. Fuson at (847) 491-3794 or fuson@nwu.eduu, or write to her at the School of Education and Social Policy, Northwestern University, 2115 North Campus Drive, Evanston, IL 60208.

Theoretical Underpinnings of the Curriculum

Our project uses a Vygotskian model for unfolding, formulating, and solving mathematics problems from children's experience. This model describes one way in which teachers build on children's prior knowledge about various situations to facilitate students' construction of understandings of formal mathematical concepts, symbolism, and problems

(see Fuson et al. [1997] for more details). The unfolding multiple narratives of different children's experiences provide a framework that is co-constructed by the teacher and children and within which teachers relate new mathematical ideas to children's lives.

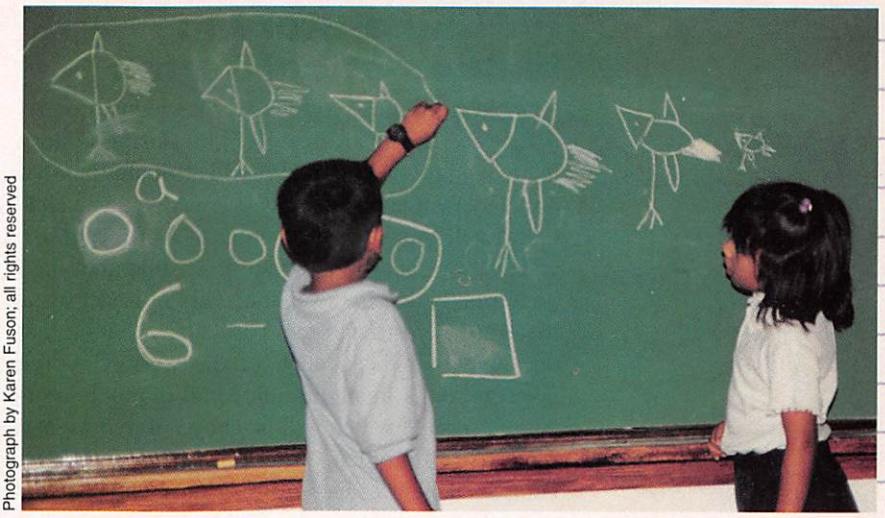
This building on children's knowledge is balanced by the other vital Vygotskian aspect of our approach: teaching within the "zone of proximal development" (ZPD). The ZPD, or learning zone, is what children can accomplish with assistance. The teacher leads children from a starting point to more advanced mathematical knowledge. This knowledge includes being better at listening, explaining and helping one another understand; learning more advanced, efficient, and accurate solution methods; and learning mathematical symbolism, language, and new ideas. The teacher, and eventually other children, help students progress in all these ways. The teacher is guided by an ambitious vision of the growth in children's knowledge by the end of the year and is supported by the CMW curriculum.

Getting Started: Eliciting and Using Children's Stories

Some children are eager and ready to share their stories. Others are initially too shy to relate their stories to the class, so they draw or write their stories. Eventually all children participate. Asking children to bring photographs from home either about a trip or any other subject can produce intriguing stories and can give teachers insights into the children's lives. Children enjoy listening to one another's stories. Since each story gives some insight into that child's life, children believe that they are a part of a dynamic class. Stories can be told at other times of the day and told again during mathematics class, perhaps by another child, to emphasize listening and remembering. Each child's story can be repeated during the year to foster ongoing coherence and inclusion. Mathematical aspects can be expanded, and nonmathematical points can be discussed in other subject areas. Children enjoy hearing their story mentioned again in class. Using children's stories in these ways involves teaching and learning processes that develop thinking and creativity and facilitate children's emerging mental, oral, and written competence with language *and* mathematics.

One classroom teacher made these comments:

> In different years, different themes have emerged from the class and from other nonmath activities we are doing. One year, we began with a story of a child whose grandmother made and sold candy in Mexico. We worked on lots of stories about packaging candy to buy and sell, made candy using the grandmother's recipe, and had a bake sale for another grade. The whole math curriculum for that year was developed around these and other stories about buying and selling. In another year in first grade, we began with a child whose dog was back in Mexico. The grandfather gives the dog five bones a day. We made many stories about how many bones Paco has had and about feeding and taking care of other animals. To enrich these stories, we may bring people from the business community, as well as family members, to talk to the students about math in their lives or their jobs.



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Understanding, Listening, and Describing

The teacher first develops the whole-class understanding of the child's story by asking other children to tell the story in their own words and to ask and answer questions about the story. This phase facilitates listening, memory, and participation as well as understanding. The teacher has the children ask questions about mathematical aspects of the story. Children become very good at asking questions about a situation. Posing the question is usually the most difficult part of writing a word problem for children, so whole-class modeling and practice in asking questions are very helpful. Less-advanced children can participate well in this phase.

One teacher describe her experience in this way:

> It has not been easy for me to move from the old paper-and-pencil math to developing a language of inquiry to support math understanding. Encouraging students to draw their problems and answers, to act out situations, to work sometimes in pairs, and to have students explain their math thinking can be a struggle. But if we want children to feel free to take risks, we have to take them, too. Frustration many times precedes insight, for children and for teachers. This task of understanding math becomes easier when we bring children's experiences into the classroom. Meaning emerges from context and connectedness.

Putting a Story in Mathematical Terms

After hearing the story, the teacher focuses on its potential mathematical aspects by posing a story that contains complex real-world attributes but that omits many nonmathematical elements. Some children retell this story in their own words and ask and answer questions about it so that the class understands this new version. This information is then further restricted to a particular situation that occurs within the context of the story. Children pose questions about different kinds of problem situations, one of which will be chosen to represent a typical word problem. After this process has been completed, the teacher can use only parts of this process on some days.

One teacher related this scenario:

> In my second-grade class, children told many stories about going to the store with their family. The children then generated several possible questions about the situation and made word problems about the situation.

Problem Solving, Reflecting, and Explaining

The class then moves into the problem-solving phase, in which children solve problems individually using their own mathematics drawings. The core of problem solving is understanding the situation. Drawing the situation engages children in this analysis. Early in first grade, children learn to make labeled drawings that show the mathematical aspects of the situation with circles or other shapes, line segments, and spacing. Labeling with letters or words ties the parts of the drawing to the situation. These drawn models help children understand the situation, reflect on their own problem-solving method, and explain their solution steps (see **fig. 1**). These explanations give teachers insight into children's mathematical thinking and help students learn from one another. These coherent, extended, and meaningful interactions engage the students and help them make connections between the mathematics concepts and language embedded in their everyday cultural practices and their emerging mathematical concepts, vocabulary, and notations.

One teacher explains the insight gained:

> While students are working, I watch the students working at the board, and I walk around listening and looking at work at children's seats. I note who has different solutions to explain and who is in trouble. Sometimes students work and discuss with a partner so that they learn from each other's thinking. Listening to their dialogues gives me an insight into their thinking and how I can expand their comprehension. When a pair of students explain their work, I ask the less-advanced student to start first. In this way, even if only at the level of description, the student feels that she or he has contributed. Other children ask about a method if they do not understand it. This is a crucial aspect that moves the math talk from being focused just on me and creates direct student-to-student interaction. After a method is described, I usually ask how many did it that way. This [tactic] increases interest, involvement, and analysis of methods—"Is mine the same or different?" We frequently discuss strengths and weaknesses of different methods. After some correct methods, I pick one or two major wrong answers to discuss so that the confusions behind them can be cleared up. Using the children's math drawings enables all children to be active listeners in the conversation.

The Co-Constructing Process

The classroom conversation is co-constructed by all those involved. The active participants in a conversation each direct the conversation in certain ways. Each contribution stimulates thinking. Throughout the conversation, personal meanings are continually being constructed and reconstructed in ways that are influenced by the classroom process. The emerging group history and climate support the sense of all participants that the conversation is a common group product created and shared by all members. All students in the class build and contribute to an environment in which everyone helps everyone else learn—sometimes actively and sometimes through emotional support—while patiently waiting for another to give input. Thus, the growing mathematical learning of each student is the result of the thinking, stories, and explanations of every student.

The teacher plays important roles in creating, maintaining, and using this sense of class mathematical history. This shared history rests on mutual respect and on explicit acknowledgment of the importance of the participation and contributions of each student. The collaborative interactions foster participants' understanding of language, drawings, notations, and mathematical conceptual structures that are common enough to permit meaningful conversations using the emerging "taken-as-shared meanings" (Cobb and Bauersfeld 1995). The teacher leads children in group and individual reflection on their personal meanings and facilitates critical thinking and decision making. These conversations flourish in classrooms that inspire learning, support children's self-regulation, build self-confidence, and provide regular feedback on learning progress.

One teacher made these notes:

> My approach to math teaching is one where students feel free to express themselves by playing an active role in the

teaching-learning process. I try to give learners enough time to assimilate and to contribute ideas. I work to create a "family" feeling of belonging to the class so that students pay attention to each other and help each other. It is important to encourage students to be autonomous, to seek meaning, to help them articulate questions, and to sense their needs. This creates a learning environment that is stimulating and tolerant, but filled with excitement to push forward.

It is also important to help students learn how to help each other. There are many of them and only one of me. In a co-constructed classroom, the class will participate in helping a student who is struggling with some concept or solution. This in-depth clarification of concepts helps everyone. Students begin to see things from another point of view. With help from the teacher, students naturally become supportive of each other, and errors are seen as an opportunity for solving and posing new problems. Such an approach allows the teacher to evaluate how and what the class has learned and how to strengthen this understanding.

Conclusion

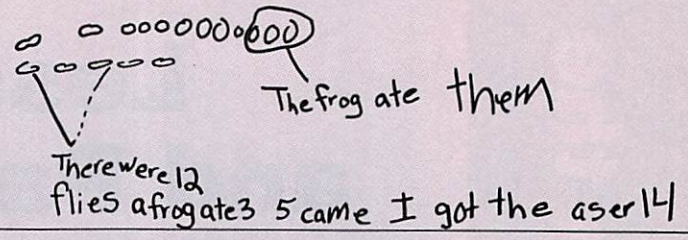
Listening to children, putting their stories in a mathematical context, using children's labeled mathematics drawings and number drawings, and eliciting explanations from children about how they solved problems are powerful approaches. But these approaches need constant leadership by the teacher so that children can progress in their knowledge of mathematical methods, vocabulary, and understanding. The CMW curriculum supports teachers in these efforts. The teaching and learning experiences are adapted to the participants and enable them to progress and become competent in mathematics.

FIGURE 1

Children's labeled mathematics drawings

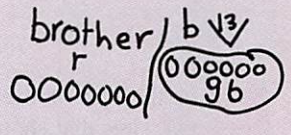
There were 12 flies in the yard. Then a frog ate 3, but later another 5 flies arrived. How many flies are there in the yard now?

14



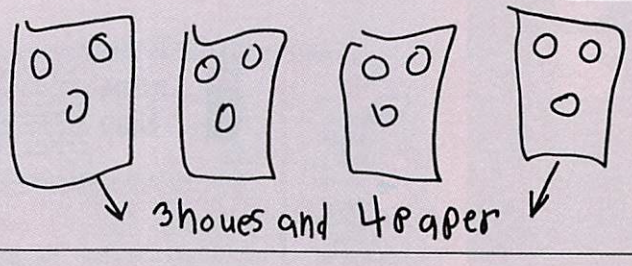
The clown gave my little brother 7 red balloons and some green balloons. Altogether my brother got 13 balloons. How many green balloons did he get?

6



I drew 3 houses on each piece of paper. I had 4 pieces of paper. How many houses did I draw in all?

12



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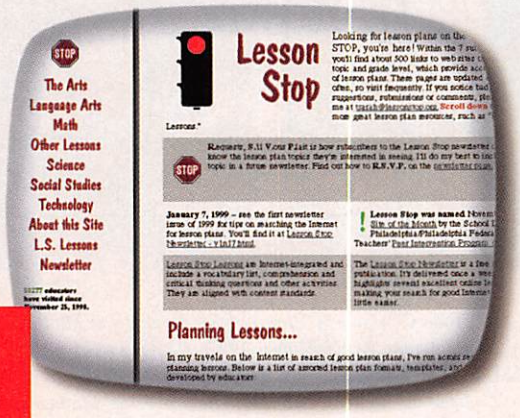
Lesson Stop and Parents Guide to the Internet

Beth Lazerick

For the end of the school year, we present two sites that should appeal to diverse populations. The Lesson Stop Web site will be of interest to teacher educators as well as teachers in search of lesson-plan ideas.

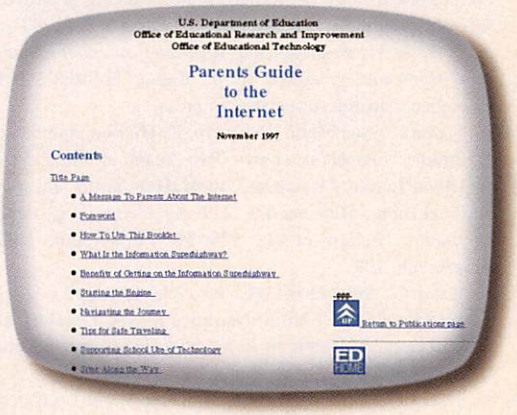


See **www.lessonstop.org** for lesson-planning tools and for help with—

1. creating lesson plans that include technology.
2. avoiding lesson-plan pitfalls.
3. finding lesson-plan templates.
4. viewing Internet-integrated lessons.

Also available are links to many other lesson-plan sites, including the Math Forum and PBS's Math Lessons and Mathline. Lesson Stop also links to numerous lesson plans for arithmetic, statistics, and geometry at the K–6 level. Teachers can sign up for a free e-mail newsletter, too.

The United States government site that provides the "Parents' Guide to the Internet" will be of interest to teachers who want to offer their students' families some compelling reading over the summer. Encourage parents to visit **www.ed.gov/pubs/parents/internet** to download and read a comprehensive guide to using the Internet at home. Teachers may want to include lists of appropriate mathematics sites from *Teaching Children Mathematics* to encourage mathematical explorations over the summer months. ▲



Edited by **Beth Lazerick,** *BethLaz@aol.com, 5691-D Fox Hollow Drive, Boca Raton, FL 33436. Readers are encouraged to send addresses for Web sites related to mathematics or mathematics education to the editor.*

The National Council of Teachers of Mathematics is not affiliated with, and has not tested the software at, the sites listed. Reference to these sites does not imply endorsement of the content or its publisher by NCTM. Since the use of items found on the Internet may cause potential problems, NCTM cautions users about the risks involved before proceeding.

Computer Materials

Fizz and Martina's Math Adventures: Blue Falls Elementary. *Gr. 3–4, CD-ROM, software and electronic teacher's guide with reproducible worksheets, $79.95. Requirements: Macintosh: System 7.1 or later, 8 MB of RAM or 16 MB of RAM with Macintosh Power PC. IBM compatible: 486 or higher with sound card; Windows 3.1 or 95, 16 MB of RAM. Tom Snyder Productions, 80 Coolidge Hill Rd., Watertown, MA 02172-2817, (800) 342-0236.*

Six titles comprise the Fizz and Martina's Math Adventures series. Each title is geared to different elementary grade levels. The software is part of a genre called Interactive Group Software. This series is designed to encourage teamwork and interaction among students but also helps develop listening skills and note taking for the individual.

Blue Falls Elementary is an adventure story containing four episodes of two parts each. To play, the class is divided into teams. Each member of each team receives a worksheet, with questions and fill-in-the-blank sentences, on which to record information from the video portion of the adventure. After each individual student records answers on this worksheet, the team members discuss and agree on the notes they have taken. These notes help them answer the questions on the next worksheet, titled Team Questions. Students must first solve the mathematics problem that was posed during the story that they have just watched. Students must work together and help each other to understand the answers that they have found. The third step is the team quiz. The questions are posed to members of the teams, and without consulting the worksheet, the chosen member must answer the questions correctly to receive award cards for her or his team.

Some follow-up activities are provided for each episode. In the first one, Trivial Computes, students answer trivia questions about the episode and then add the numbers in the responses to find a given magic number. This activity contradicts the claim of the authors that "[u]nlike typical story problems, each math problem emerges naturally within a compelling narrative context and has dramatic consequences" (p. 5).

The second activity, The Estimation Game, encourages students to estimate and then to explain the strategy that they have used. A successful strategy can also be explained by a click on a button. The three estimation problems presented in each episode are very good examples for students at this level.

The third activity, Homework, displays a list of optional homework activities. Practice problems and a Write-Your-Own-Story-Problem worksheet are included for each episode.

Overall, I think this software is a fine instrument to develop cooperative learning in the classroom. Because I reviewed this product in the summer, I did not test it in the classroom; however, it appears to be interesting and stimulating enough to engage most children.

A few drawbacks were found. First, the activity is very time-consuming. Each episode is forty-five minutes long, and some follow-up activities must immediately follow the episode. Second, students must take notes as they view the episode. Some of the information is presented rather rapidly, which makes it likely that students will miss some pertinent information. Another drawback is the price. Each episode can be used only once each year.

This software seeks to develop a cooperative-learning environment and can be used to this end very successfully. Mathematics, as arithmetic only, is integrated into the story; and the types of questions encourage mathematical talk and mathematical problem solving. The issues of time and cost would be my only cautions to teachers who are considering the purchase of this package.—*Barbara A. Burns, State University of New York at Buffalo, Amherst, NY 14260.*

Incredible Tutor: Primary Math. *Ver. 2.5 CD-ROM, floppy key disk, and manual, $40. Requirements: Windows 3.1, 386 processor, 6 MB of RAM; Macintosh: System 7.1, 68030 processor, 6 MB of*

Edited by **Douglas Clements,** *State University of New York at Buffalo, Buffalo, NY 14260*

Prices on software, books, and materials are subject to change. Consult the suppliers for the current prices. The comments reflect the reviewers' opinions and do not imply endorsement by the National Council of Teachers of Mathematics.

RAM. Sleek Software Corp., 2404 Rutland Dr., Ste. 600, Austin, TX 78758, (800) 337-5335.

Incredible Tutor is designed to help students learn skills in various subject-matter areas and grades. This review was conducted on the grades 3–5 mathematics activities. The software is intended to be used in a classroom or computer laboratory by individual students.

The installation instructions in the manual are neither polished nor simple. For example, all software from this company is installed from the same CDs, but only those pieces purchased may be selected and used. A key must be entered or a floppy disk inserted. Neither the manual nor the software screens are teacher-friendly. Nevertheless, the software did install successfully. Overall, the manual does not focus much on educational issues.

The software presents mathematics problems covering a wide variety of topics. The problems are presented in a problem window. The student must conduct all interactions by using a row of buttons in a thin panel on the left side of the screen. Interactions are for the most part limited to choosing multiple-choice answers and navigating through the program. The student can view solved problems, although the solutions presented tend to be very wordy.

The strength of the program lies in the area of management. The company sells a related teacher-utility program that can be used to manage and report on many students and classes at one time. However, elements of a management system are included in Incredible Tutor. This feature may be an advantage to some teachers, although the dialogue boxes for setting up various operations may be daunting, and some screens are not consistent with normal computer-interface expectations.

In summary, this package provides a wide variety of practice items and gives limited feedback and instruction, with the potential for significant computer-aided management. However, the rigid format and limited nature of the interactions are areas of serious concern. Given that alternatives are available and no data on the effectiveness of this program are provided, I do not recommend this software.—*D. C.*

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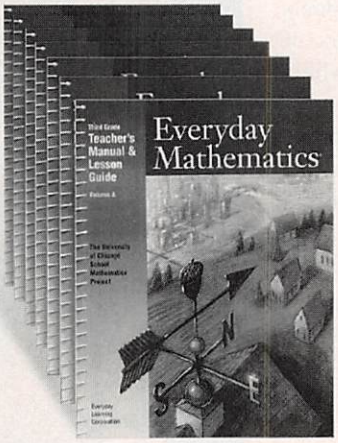

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For Students

Counting Book, *photographs by Dave King, 1998. 32 pp., $12.95 cloth. ISBN 0-7894-3448-2. Published by DK Publishing, distrib. by Publisher Resources, 1224 Heil Quaker Dr., LaVergne, TN 37086, (888) 342-5357.*

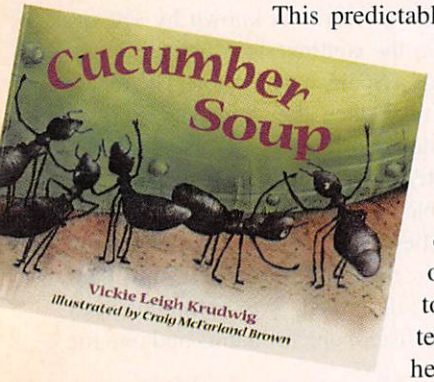
Through photographs of everyday objects, readers are introduced to the numbers 1 through 20, 30 through 100, and then 1000. Although the stated intent of the book is to "use enticing photographs and stimulating questions," the result is a book that would be too confusing for young readers. Questions run around the four corners of each page, and the ones that run along the gutter are dif-

Reviewed by **David J. Whitin,** *University of South Carolina, Columbia, SC 29208*

ficult to read. The primary problem with the book is that it attempts to do too much.

Readers are asked to count sets, count subsets, compare sets, name colors, count backward, tell time equivalences, and so on. Some questions could be confusing for young readers because they contain two numbers (e.g., "Find the number 5 on 3 presents"). No predictable order is observed to the questions that are being asked.—*D. J. W.*

Cucumber Soup, *Vickie Leigh Krudwig, illus. by Craig McFarland Brown, 1998. 32 pp. ISBN 1-55591-380-6.* **Sopa De Pepino**, *Spanish edition, trans. by Antonio Madrigal. ISBN 1-55591-982-0. $16.95 ea. cloth. Fulcrum Publishing, 350 Indiana St., Ste. 350, Golden, CO 80401, (800) 992-2908.*



This predictable story for pre-K–1 students is a variation of the folktale about the enormous turnip. In this story, a cucumber falls on the entrance to an ant hill. The ten ants enlist the help of other insects to remove this big obstacle from their doorway. Nine mosquitoes, then eight ladybugs, seven spiders, six bumblebees, and so on, arrive to help, until one flea remains. Of course, it is the flea's small weight that is enough to solve the problem.

The illustrations on each page show all the previously mentioned insects, as well as the new set of insects. At the end of the story, all the insects are lined up in a horizontal row, and their placement is certainly an invitation to count the total number of helpers. The author has included on each page a few facts about each insect as it is introduced. Some of these insects are helpful and some are harmful to a vegetable garden. The book might prompt some further research about insects. This book is also available in a Spanish edition.—*D. J. W.*

More M&M's Brand, Chocolate Candies Math, *Barbara Barbieri McGrath, illus. by Roger Glass, 1998. 32 pp., $6.95 paper. ISBN 0-88106-994-9. Charlesbridge Publishing, 85 Main St., Watertown, MA 02472, (800) 225-3214.*

McGrath has created another M&M's book for children in grades 2–4. Teachers introducing multiplication and division would find this book helpful. The author uses M&M's in a rhyming format to sort, graph, multiply, divide, and teach ordinal numbers. The brightly colored candies are arranged in clear sets against a white background. The graphing of colors introduces readers to the vocabulary of comparisons: most, least, and equal. Commutativity in multiplication is effectively conveyed by showing ten sets of 2 and two sets of 10. However, it would have been helpful to have shown these problems on facing pages so that readers could more easily have compared them. Also, four sets of 10 and five sets of 8 would have been more interesting to compare if they had been juxtaposed rather than separated by several pages. Despite some of these organizational details, the book

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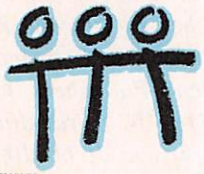

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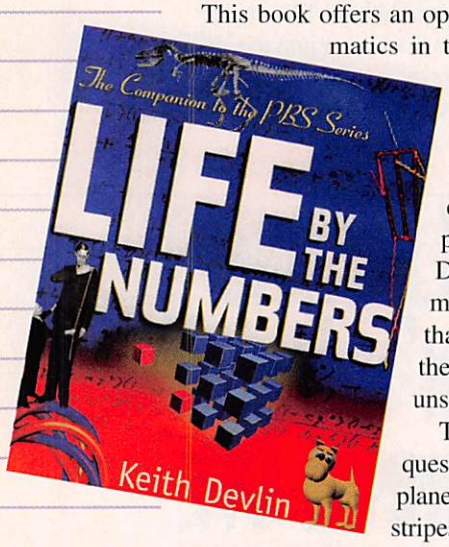
serves as an inviting introduction to these various mathematical topics.—*D. J. W.*

For Teachers

Life by the Numbers, *Keith Devlin, 1998. ix + 214 pp., $29.95 cloth. ISBN 0-471-24044-3. John Wiley & Sons, Order Processing Dept., 432 Elizabeth Ave., Somerset, NJ 08875, (800) 225-5945.*

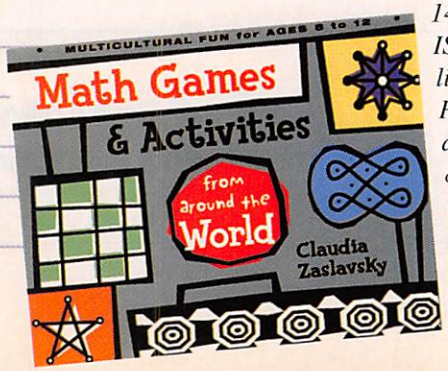


This book offers an opportunity to look at mathematics in the real world and explore how it is intertwined in our everyday lives. Devlin demonstrates that mathematics is more than the drill and practice that many people have experienced. Devlin shows that mathematics is a science and a tool that is the basis for exploring the universe and understanding unseen phenomenon.

The author explores such questions as "How does an airplane fly?" "Why do zebras have stripes?" and "How can the floor of the ocean be mapped?" Interviews with scientists and mathematicians who are currently working on problems are included in each discussion. The author presents examples of careers and professions that use mathematics as a foundation. He gives his readers historical examples of mathematical discoveries and pays tribute to the mathematicians who have made the contributions. Unfortunately, in his effort to make the subject simple and friendly, Devlin falls short of using mathematics in the topics that he explores.

Life by the Numbers would be a welcome addition to any teacher's library. Devlin offers educators a resource in their continuing effort to answer the question "How does math apply to me?" Devlin also gives educators ideas that can serve as springboards for integrated investigations in mathematics and science.—*Diane McElwain, U. L. Light Middle School, Barberton City Schools, Barberton, OH 44203.*

Math Games and Activities from around the World, *Claudia Zaslavsky, 1998. Ages 8–12, ix + 146 pp., $14.95 paper. ISBN 1-55652-287-8. Published by Chicago Review Press, distrib. by Independent Publishers Group, 814 N. Franklin St., Chicago, IL 60610, (800) 888-4741.*



In this book, noted ethnomathematician Claudia Zaslavsky presents a collection of games from many cultures worldwide. Similar games are grouped under common chapter headings, such as "Three in a Row," "Board Games," "Puzzles with Numbers," and "Puzzles without Numbers." She earmarks two chapters for geometry and other visual patterns.

Each game includes its cultural origin, materials, and directions for making and playing. Alternative ways of playing and strategies are provided when appropriate. Many of the games originated in Africa, Europe, or southeast Asia. Games from the southwestern United States are also included. Social studies teachers will find this book to be a welcome resource to supplement their instruction.

One game, called Picaria, is known by several different names in the southwestern United States and apparently has a Spanish derivation. The materials needed to reproduce the game, as well as directions, are included. Zaslavsky also includes techniques for altering the rules to make additional situations or problems to solve.

All the activities included are inexpensive to reproduce, both in time and materials. More than seventy games are described. Two comprehensive bibliographies are listed, one for adults and one for students.

The price of the book is quite reasonable for such an extensive addition to a professional library. The book is intended for ages 8–12, but many teachers will find the activities to be of interest to older students.—*Michael P. Risser, Findlay City Schools, Findlay, OH 45840.*

Middle-Grade Teachers' Mathematical Knowledge and Its Relationship to Instruction: A Research Monograph, *Judith T. Sowder, Randolph A. Philipp, Barbara E. Armstrong, and Bonnie P. Schappelle, 1998. 227 pp., $21.95 paper. ISBN 0-7914-3842-2. SUNY State University of New York Press, State University Plaza, Albany, NY 12246, (800) 666-2211.*

This research report reveals effects on middle school teachers' mathematics instruction when they are given opportunities to explore more deeply conceptual meanings of such topics as rational-number operations. Teachers in the study believed that they were "letting go" of direct instruction and leading learners to recognize concepts on their own. The major strength of the monograph is in the conclusions that teachers formulated. Because teachers had experienced the frustration of understanding rational-number operations over merely performing algo-

rithms, they began to see how students learn instead of focusing on how children should be taught. Teaching conceptually involved more than supplying manipulatives or activities. Teachers expressed the need for all mathematics teachers to have similar experiences.

Graduate students in mathematics education who are considering research topics should find this book to be helpful. Those who teach material on fractions would find this book to be a good resource on that topic and, more generally, for their professional development. The chapters that are case studies of teachers struggling with mathematical ideas could help preservice teachers in mathematics-methods courses. Conceptual thinking about ideas considered to be trivial is often raised to a new level in these courses.

In short, this volume is an excellent companion to the earlier publication *Providing a Foundation for Teaching Mathematics in the Middle Grades* (Sowder and Schappelle [Albany, N.Y.: SUNY Press, 1995]).—*Kimberly C. Arp and Charles E. Lamb, Texas A&M University, College Station, TX 77843-4232.*

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This package is designed to help intermediate and middle school students visualize number relationships and number theory through the use of three-dimensional factor blocks. The package includes an assortment of fifteen different sizes of foam blocks that show factors of numbers. For example, the $2 \times 2 \times 3$ block and the $1 \times 6 \times 2$ block show two different ways to factor the number 12. The materials include handy storage bags for each individual set of blocks.

This package is appealing in that students play with and explore number concepts. The materials could be useful in whole-class activities or as part of a mathematics center to allow students to explore a concept individually before instruction. The print materials include eighteen blackline masters for student activities and for assessment activities. The topics explored include prime numbers, greatest common factor, least common multiple, connections between GCF and LCM, and so on. The materials also include investigation evaluations that are modeled after real-life problem situations.

The blocks are made of ETA's ManipuLite material, which makes them an excellent and quiet manipulative. The only weakness that I found with these materials was the restriction to relatively small numbers. The largest number represented is 30, with a $2 \times 3 \times 5$ block.—*Eleanor Ennis, Charles Chapman Elementary School, Salisbury, MD 21801, and Tammy Burke, student at Salisbury State University.* ▲

This package is appealing in that students play with and explore number concepts.

Factor Blocks Deluxe Comprehensive Set

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Thanks from the Editorial Panel

The quality of *Teaching Children Mathematics (TCM)* very much depends on the voluntary efforts of many mathematics educators. The journal has profited from the experience of those who have served as members of the Editorial Panel, department editors, members of Reader Advisory Panels, reviewers, and referees.

Members of the Editorial Panel are appointed by the NCTM President and are approved by the Board of Directors. In April 1999, Charles Geer and DeAnn Huinker completed their terms. Geer deserves a special commendation for his leadership in producing the February 1999 focus issue, titled "Geometry and Geometric Thinking," and Huinker deserves recognition for her exemplary leadership as chair of the Panel for 1997–1999.

The importance of department editors to the quality of this journal is impossible to overstate. As editor of the "Investigations" department, Marty Hopkins provided exploration activities for direct use with students. Donald Chambers and Anne Reynolds gathered another collection of "Research into Practice" articles that were well received by classroom teachers, as well as the research community. Tad Watanabe served as the editor of "Promising Research, Programs, and Projects," and Kate Kline renewed the popular "Early Childhood Corner" department. Judith Seidel was the editor of "Tech Time," and Beth Lazerick helped readers find their way around the Internet as editor of "News from the Net." Karen Holly and Sharon Rak supplied a year of activities through the lively "Math by the Month" department. "Links to Literature" continued to be a favorite department of our readers; thanks go to editors Jacquelin Smith and Jacqueline Harris for soliciting and editing such quality contributions.

The coordination of reviews of computer materials has been handled by Doug Clements and Paul Becher. The reviews of children's books have been contributed by David Whitin.

Each article that appears in *Teaching Children Mathematics* is judged for content and style by at least three referees. Reviewers are volunteers who agree to evaluate books and other instructional materials for the journal. These volunteers serve open-ended terms. The names of NCTM members who were serving in these roles as of 1 December 1998 appear on the following list. An asterisk beside an individual's name indicates service as both a referee and a reviewer. NCTM members who are interested in refereeing two or three manuscripts each year or serving as a reviewer of instructional materials should request a copy of the guidelines and a personal data sheet from the journal editor.

The efforts of all these individuals in maintaining the vitality of *Teaching Children Mathematics* are very much appreciated.

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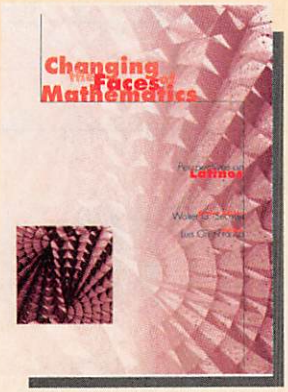

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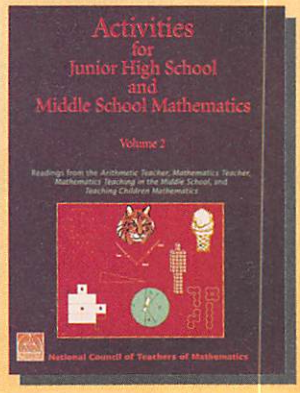

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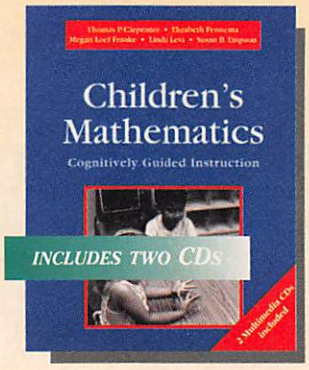

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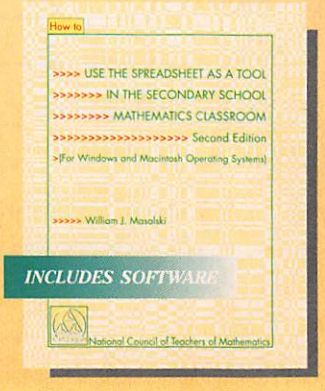

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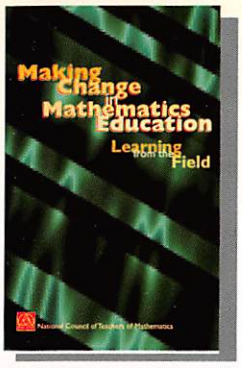

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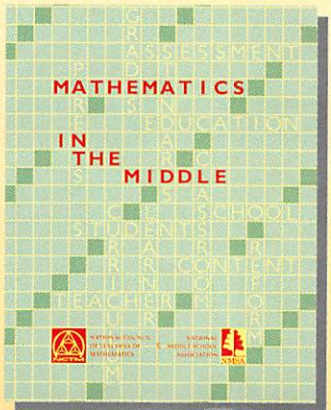

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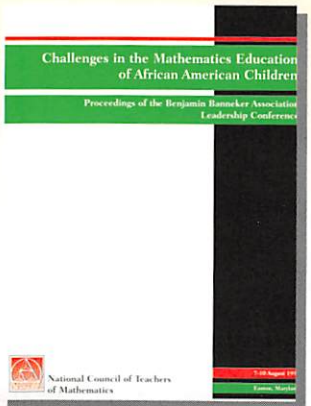

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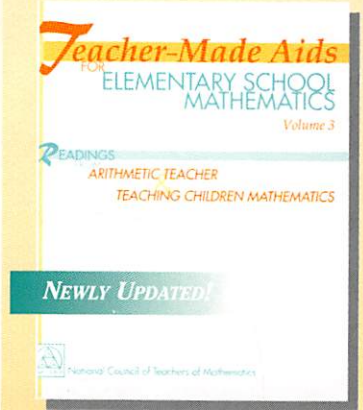

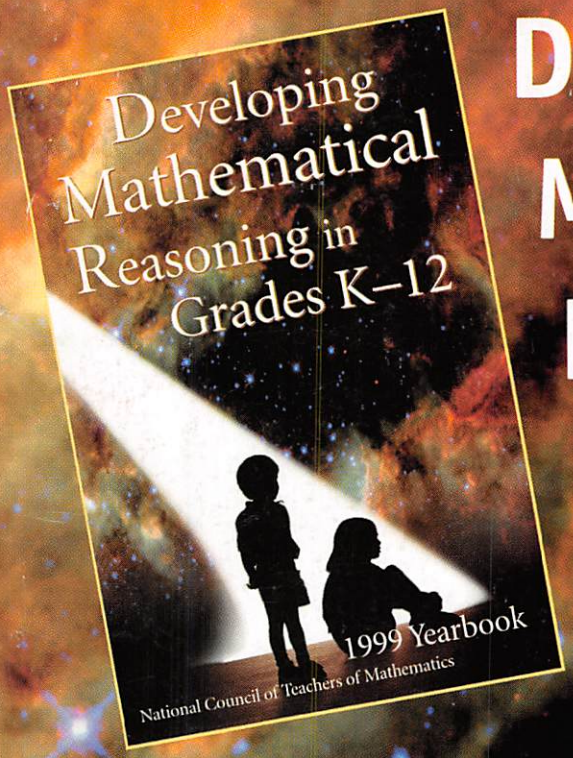

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